GEODETIC COMPUTATIONS ON A PROJECTION PLANE

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GEODETIC COMPUTATIONS ON A PROJECTION PLANE

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By

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CHAPTER I

INTRODUCTION

Since man first realized that the earth was not a plane, his attempts to make computations on it have become more and more complicated with the passing of time. For a time, he considered the earth as a sphere. This had the effect of complicating his work, but the complications were relatively minor. Later he found that the earth was not a sphere, but more nearly an ellipsoid of revolution. He occupied himself for hundreds of years trying to find the parameters of this ellipsoid. With each new measurement came the possibility of new values for the parameters of this ellipsoid.

Computations on an ellipsoid are complicated. Most formulas are expressed in a series development, with each successive term of the series becoming increasingly complicated. The greater the distance to be computed, the greater the number of terms required to give a consistent degree of accuracy. There is no doubt that the computations made on a reference ellipsoid are correct, if a sufficient number of terms is considered. The fact that the location of the reference ellipsoid, with respect to any measurements made on the earth, is unknown, does not invalidate the purity of the mathematical processes, even though the geodetic positions and distances may be incorrect.

The problem to be considered in this paper is whether or not the complicated computations on an ellipsoid are warranted

in all cases, or whether simplified methods could be used for some work. This investigation considers geodetic computations on a projection plane, determining the limits within which plane coordinates may be used.

CHAPTER II

TERMINOLOGY AND FORMULAS

Geodesy is that branch of surveying which takes into account the curvature of the earth. The most common method of making geodetic computations is to compute approximate triangles from spherical angles in order to find the spherical excess, then, after allowing for spherical excess, correcting the observed angles by some provisional method, and computing lengths of sides of the preliminary triangles by the sine law. With these lengths, new positions are found by computing on a sphere whose radius is equal to the radius of curvature in the prime vertical at the initial point of each line. To the value obtained on the sphere is added a small correction which accounts for the difference between computing on the sphere and on the ellipsoid. This procedure yields preliminary positions. Using information computed during the preliminary position computation, conditions are imposed on the net, and a least squares adjustment of the triangulation is made which yields corrections to the observed directions. After these corrections are applied, it is necessary to recompute the triangles, and recompute the positions.

The method of making computations to be investigated in this paper is to work on an orthomorphic projection plane with plane coordinates. A Transverse Mercator projection will be used, with specifications conforming to the Universal Trans-

verse Mercator Grid system. Corrections will be added to observed directions to convert them to plane directions, preliminary positions will be determined by an intersection formula, and final positions will be obtained by a least squares adjustment. The corrections which convert from observed directions to plane directions are functions of the curvature of the earth, so the computations may be called geodetic, even though performed on a plane.

The following symbols will be used ——

φ - Latitude, positive northward, measured from equator.

λ - Longitude, positive eastward, measured from central meridian of the projection.

ξ - Plane coordinate in a generally north direction, positive to the northward, measured from the equator.

η - Plane coordinate in a generally eastward direction, positive to the eastward, measured from the central meridian of the projection.

α - Azimuth of a line, measured clockwise from the south.

T - Grid direction of the depiction of the geodesic on the projection plane, measured clockwise from the positive ξ axis.

t - Grid direction of the rectilinear chord joining two points on the projection plane, measured clockwise from the positive ξ axis.

c - meridian convergence, measured from the meridian to the ξ axis, positive if clockwise (point east of the

central meridian), negative if counterclockwise (point west of the central meridian).

M - Meridional distance from equator, positive northward.

s - Length of line on projection.

S - Length of line on ellipsoid.

m - Scale factor = $\lim \frac{s}{S}$ as S goes to zero.

m_o - Scale factor along the central meridian.

ξ_r - $m_o M = f(\varphi)$, whereas $\xi = f(\varphi,\lambda)$.

R - Radius of curvature in the meridian.

N - Radius of curvature in the prime vertical.

r - Mean radius of curvature = $(RN)^{1/2}$.

e^2 - Square of the first eccentricity of the meridian.

e'^2 - Square of the second eccentricity of the meridian.

ρ° - 57.295 77951 degrees per radian.

The specifications for the Universal Transverse Mercator Grid system necessary for this study are ——

1. Projection graticule is the Transverse Mercator.
2. Origin of grid coordinates is the intersection of the central meridian of each zone with the equator.
3. Unit of measurement is the meter.
4. Northing equals ξ in the northern hemisphere, ξ plus 10,000,000 in the southern hemisphere.
5. Easting equals 500,000 plus η.
6. The scale factor along the central meridian is 0.9996.
7. Grid zones run from 80° south to 80° north, and are bounded by meridians which are multiples of 6°.

The principle behind the Transverse Mercator Projection is:

1. The reference ellipsoid is mapped conformally on a sphere.
2. The sphere is divided into tesserae by transverse meridians which run through poles on the equator whose longitude is that of the central meridian plus and minus 90°, and by transverse parallels, which intersect the transverse meridians at an angle of 90°.
3. The transverse meridians and parallels are developed in accordance with Mercator's principles for a loxodromic, conformal projection, the meridians becoming lines of constant ξ, while the parallels become lines of constant η.

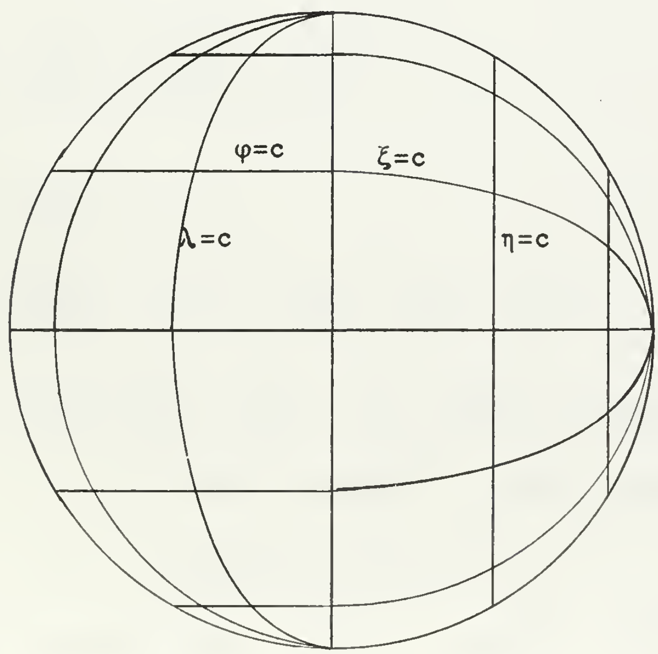

Fig. 1

It has become standard practice, in many mapping agencies, to convert geographic coordinates to Universal Transverse Mercator Grid coordinates as soon as the geographic coordinates are determined, since it is in the rectangular form that the positions will ultimately be used. If the positions were to be computed directly in the rectangular system, it would not be necessary to convert to geographic coordinates en masse, since, in the majority of cases, the latitude and longitude are not desired, but the point is used only as control for graphic processes or for further computation.

Thomas (6) has developed the following formulas dealing with the transformation from geographic to grid coordinates and back, with convergence of the meridians, and with scale factor ——

$$\underline{\xi = \xi_r + A_2\lambda^2 + A_4\lambda^4 + A_6\lambda^6 + A_8\lambda^8,} \qquad \text{(A)}$$

where

$A_2 = m_oN \sin\varphi \cos\varphi \,/\, 2\rho^2$,

$A_4 = m_oN \sin\varphi \cos^3\varphi \,(5 - z^2 + 9n^2 + 4n^4) \,/\, 24\rho^4$,

$$A_6 = m_oN \sin\varphi \cos^5\varphi \,(61 - 58z^2 + z^4 + 270n^2 - 330z^2n^2 + 445n^4 + 324n^6 - 680n^4z^2 + 88n^8 - 600n^6z^2 - 192n^8z^2) \,/\, 720\rho^6 ,$$

$A_8 = m_oN \sin\varphi \cos^7\varphi \,(1{,}385 - 3{,}111z^2 + 543z^4 - z^6) \,/\, 40{,}320\rho^8$,

$z = \tan\varphi$, and

$n^2 = e'^2 \cos^2 \varphi .$

$$\underline{\eta \;= B_1 \lambda + B_3 \lambda^3 + B_5 \lambda^5 + B_7 \lambda^7} \;, \qquad \text{(B)}$$

where

$B_1 = m_o N \cos \varphi \,/\, \rho,$

$B_3 = m_o N \cos^3 \varphi \,(1 - z^2 + n^2) \,/\, 6\rho^3 ,$

$B_5 = m_o N \cos^5 \varphi \,(5 - 18z^2 + z^4 + 14n^2 - 58z^2 n^2 + 13n^4 + 4n^6$

$\qquad - 64n^4 z^2 - 24n^6 z^2) \,/\, 120\rho^5$, and

$B_7 = m_o N \cos^7 \varphi \,(61 - 479z^2 + 179z^4 - z^6) \,/\, 5{,}040\rho^7.$

$$\underline{c \;= C_1 \lambda + C_3 \lambda^3 + C_5 \lambda^5 + C_7 \lambda^7}, \qquad \text{(C)}$$

where

$C_1 = \sin \varphi,$

$C_3 = z \cos^3 \varphi \,(1 + 3n^2 + 2n^4) \,/\, 3\rho^2 ,$

$C_5 = z \cos^5 \varphi \,(2 - z^2 + 15n^2 + 35n^4 - 15n^2 z^2 + 33n^6 - 50n^4 z^2$

$\qquad + 11\, n^8 - 60z^2 n^6 - 24z^2 n^8) \,/\, 15\rho^4$, and

$C_7 = z \cos^7 \varphi \,(17 - 26z^2 + 2z^4) \,/\, 315\rho^6.$

For the inverse problem it is necessary to find a value φ_1 which corresponds to $\xi_1 = \xi$, $\eta_1 = 0$, and compute parameters such as z, n, R, N, etc. at φ_1.

$$\underline{\varphi \;= \varphi_1 - D_2 \eta^2 + D_4 \eta^4 - D_6 \eta^6 + D_8 \eta^8} \;, \qquad \text{(D)}$$

where

$$D_2 = z_1 \rho / \ 2\, m_o^{\ 2} R_1 N_1 \ ,$$

$$D_4 = z_1 \rho (5 + 3z_1^2 + n_1^2 - 4n_1^4 - 9n_1^2 z_1^2) \ / \ 24\, m_o^4 R_1 N_1^3 \ ,$$

$$D_6 = z_1 \rho (61 + 90z_1^2 + 46n_1^2 + 45z_1^4 - 252z_1^2 n_1^2 - 3n_1^4 + 100n_1^6$$
$$- 66z_1^2 n_1^4 - 90z_1^4 n_1^2 + 88n_1^8 + 225z_1^4 n_1^4 + 84z_1^2 n_1^6$$
$$- 192z_1^2 n_1^8) / \ 720\, m_o^6 R_1 N_1^5, \text{ and}$$

$$D_8 = z_1 \rho (1{,}385 + 3{,}633z_1^2 + 4{,}095z_1^4 + 1{,}574z_1^6) \ / \ 40{,}320\, m_o^8 R_1 N_1^7 \ .$$

$$\underline{\lambda \ = E_1 \eta - E_3 \eta^3 + E_5 \eta^5 - E_7 \eta^7} \ , \qquad \text{(E)}$$

where

$$E_1 = \rho / \ m_o N_1 \cos \varphi_1 \ ,$$

$$E_3 = \rho \ (1 + 2z_1^2 + n_1^2) \ / \ 6\, m_o^3 N_1^3 \cos \varphi_1 \ ,$$

$$E_5 = \rho \ (5 + 6n_1^2 + 28z_1^2 - 3n_1^4 + 8z_1^2 n_1^2 + 24z_1^4 - 4n_1^6 + 4z_1^2 n_1^4$$
$$+ 24z_1^2 n_1^6) \ / \ 120\, m_o^5 N_1^5 \cos \varphi_1 \ , \text{ and}$$

$$E_7 = \rho (61 + 662z_1^2 + 1{,}320z_1^4 + 720z_1^6) \ / \ 5{,}040\, m_o^7 N_1^7 \cos \varphi_1 \ .$$

$$\underline{c \ = F_1 \eta - F_3 \eta^3 + F_5 \eta^5 - F_7 \eta^7} \ , \qquad \text{(F)}$$

where

$$F_1 = z_1 \rho / \ m_o N_1 \ ,$$

$$F_3 = z_1 \rho \ (1 + z_1^2 - n_1^2 - 2n_1^4) \ / \ 3\, m_o^3 N_1^3 \ ,$$

$$F_5 = z_1 \rho (2 + 5z_1^2 + 2n_1^2 + 3z_1^4 + z_1^2 n_1^2 + 9n_1^4 + 20n_1^6 - 7z_1^2 n_1^4$$
$$- 27z_1^2 n_1^6 + 11n_1^8 - 24z_1^2 n_1^8) \ / \ 15\, m_o^5 N_1^5 \ , \text{ and}$$

$$F_7 = z_1 \rho \ (17 + 77z_1^2 + 105z_1^4 + 45z_1^6) \ / \ 315\, m_o^7 N_1^7 \ .$$

$$\underline{m = m_o + G_2\lambda^2 + G_4\lambda^4 + G_6\lambda^6,} \tag{G}$$

where

$G_2 = m_o \cos^2\varphi\ (1 + n^2) \ / \ 2\rho^2$,

$G_4 = m_o \cos^4\varphi\ (5 - 4z^2 + 14n^2 + 13n^4 - 28z^2n^2 + 4n^6 - 48z^2n^4$
$\qquad - 24z^2n^6) \ / \ 24\rho^4$, and

$G_6 = m_o \cos^6\varphi\ (61 - 148z^2 + 16z^4) \ / \ 720\rho^6$.

$$\underline{m = m_o(1 + H_2\eta^2 + H_4\eta^4 + H_6\eta^6) ,} \tag{H}$$

where

$H_2 = (1 + n_1^2) \ / \ 2m_o^2N_1^2 \ = \ 1 \ / \ 2m_o^2r_1^2$,

$H_4 = (1 + 6n_1^2 + 9n_1^4 + 4n_1^6 - 24z_1^2n_1^4 - 24z_1^2n_1^6) \ / \ 24m_o^4N_1^4$, and

$H_6 = 1 \ / \ 720m_o^6N_1^6$.

The above formulas are applicable only to point-to-point coordinate transformation, and cannot be applied to a line on the projection plane. When a geodesic, other than the equator or the central meridian, is depicted on the projection plane, it is not a straight line, but a curve which is concave towards the central meridian. In order to work on the plane it is necessary to find the difference between the grid direction of the depiction of the geodesic and the grid direction of the rectilinear chord between the two terminals of the line.

Since a triangle on the ellipsoid has spherical excess, and the equivalent triangle on the projection plane has none,

it is obvious that the correction between geodesic and rectilinear chord must compensate for the effect of spherical excess. This correction is called the T-t correction (Fig. 2). Hotine (3) has developed this correction as ——

$$(T-t)_1 = I_1(\xi_2-\xi_1)\eta_3 + I_2(\eta_2-\eta_1)\eta_3^2 - I_3(\xi_2-\xi_1)\eta_3^3 - I_4(\eta_2-\eta_1)\eta_3^4 \,, \qquad \text{(I)}$$

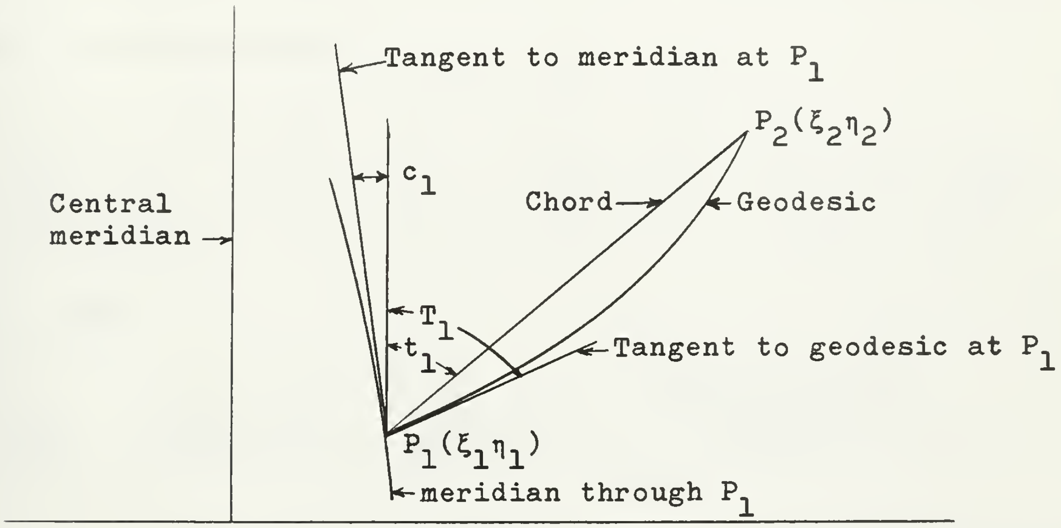

Fig. 2

where

$$I_1 = \rho \,/\, 2m_o^2R_3N_3 \,,$$

$$I_2 = \rho z_3n_3^2 \,/\, m_o^3R_3N_3^2 \,,$$

$$I_3 = \rho(1 - n_3^2) \,/\, 6m_o^4R_3N_3^3 \,,$$

$$I_4 = \rho z_3n_3^2 \,/\, 6m_o^5R_3^2N_3^3 \quad , \text{ and}$$

the subscript 3 indicates values at one-third of the way from P_1 to P_2 along the rectilinear chord, which means that coefficients

should be determined for $\xi_r = \frac{1}{3}(2\xi_{r1} + \xi_{r2})$, or for a value of φ corresponding to $M = \frac{1}{3}(2M_1 + M_2)$.

The above formulas are sufficient for all computations when working from or between known points. If, however, a line of known length on the ellipsoid is to be used, such as in the case of a base line or in trilateration, it is necessary to change the ellipsoidal length to conform to the varying scale factor on the projection plane.

$m = m_o(1 + H_2\eta^2 + H_4\eta^4 + H_6\eta^6)$, but $m = \frac{ds}{dS}$, so

$$S = \int_0^s \frac{ds}{m} = \int_0^s \frac{ds}{m_o} (1 - H_2\eta^2 - H_4\eta^4 - H_6\eta^6 + H_2^2\eta^4 + 2H_2H_4\eta^6 - H_2^3\eta^6) ,$$

$$ds = \frac{d\eta}{\sin t} ,$$

$$S = \frac{s}{m_o} - \int_{\eta_1}^{\eta_2} \frac{H_2}{m_o \sin t}\eta^2 d\eta + \int_{\eta_1}^{\eta_2} \frac{(H_2^2-H_4)}{m_o \sin t}\eta^4 d\eta - \int_{\eta_1}^{\eta_2} \frac{(H_2^3-2H_2H_4+H_6)}{m_o \sin t}\eta^6 d\eta ,$$

$$= \frac{s}{m_o} - \frac{H_2}{3m_o \sin t}(\eta_2^3-\eta_1^3) + \frac{(H_2^2-H_4)}{5m_o \sin t}(\eta_2^5-\eta_1^5) - \frac{(H_2^3-2H_2H_4+H_6)}{7m_o \sin t}(\eta_2^7-\eta_1^7),$$

but $\frac{1}{\sin t} = \frac{s}{\eta_2-\eta_1}$, so

$$S = \frac{s}{m_o}\left(1 - \frac{H_2}{3} x \frac{\eta_2^3-\eta_1^3}{\eta_2-\eta_1} + \frac{H_2^2-H_4}{5} x \frac{\eta_2^5-\eta_1^5}{\eta_2-\eta_1} - \frac{H_2^3-2H_2H_4+H_6}{7} x \frac{\eta_2^7-\eta_1^7}{\eta_2-\eta_1}\right), \text{ or}$$

$$S = \frac{s}{m_o} (1 - J_2(\eta_1^2+\eta_1\eta_2+\eta_2^2) + J_4(\eta_1^4+\eta_1^3\eta_2+\eta_1^2\eta_2^2+\eta_1\eta_2^3+\eta_2^4)$$
$$- J_6(\eta_1^6+\eta_1^5\eta_2+\eta_1^4\eta_2^2+\eta_1^3\eta_2^3+\eta_1^2\eta_2^4+\eta_1\eta_2^5+\eta_2^6)) , \qquad (J)$$

---

where

$J_2 = H_2 / 3$,

$J_4 = (H_2^2 - H_4) / 5$, and

$J_6 = (H_2^3 - 2 H_2 H_4 + H_6) / 7.$

The constants J_2, J_4, and J_6 must be determined for the mean footpoint latitude, where the footpoint latitude is that latitude corresponding to the given value of ξ if $\eta = 0$. In terms of rectangular coordinates, the constants will be determined for the mean value of ξ.

Equation J is quite unwieldy to use, becoming increasingly difficult as η increases. As a substitute, Tardi and Laclavere (5) have suggested using the following formula, where the subscripts 1 and 2 indicate the terminals of the line, and ½ indicates the mid-point —

$$\left(\frac{1}{m}\right)_{avg} = \frac{1}{6}\left(\frac{1}{m_1} + \frac{4}{m_{1/2}} + \frac{1}{m_2}\right) . \qquad (K)$$

The scale obtained by use of this formula will be compared with that obtained by Equation J.

Equations A through K constitute all the equations necessary for transforming observations back and forth between the ellipsoid and the projection plane. These formulas are quite complicated, and it will be necessary to determine how many of the terms may be excluded without adversely affecting the accuracy of the computations, and to arrange the remaining terms in tabular or graphical format so that the equations may be used without undue difficulty.

CHAPTER III

INVESTIGATION

Before Using Equations A through K, it is necessary to determine the accuracy to be sought after, and to find the limits within which the equations may be used.

In geodetic computations, latitude and longitude are carried to thousandths of a second. For latitude, 0.001 seconds is equivalent to three centimeters, while for longitude, 0.001 seconds is equivalent to three centimeters multiplied by the cosine of the latitude, so it varies from three centimeters at the equator to slightly more than five millimeters at $\varphi = 80^\circ$, equalling one centimeter at about $\varphi = 70^\circ$. Consequently, computations will be carried out to millimeters, but positions will be rounded off to centimeters. For the inverse problem, positions will be computed in degrees and decimals in order to avoid the difficulties of working with the sexagesimal system. The one-thousandth part of a degree, which is equivalent to 3.6 seconds, will be referred to as a milli-degree, while the one-millionth part of a degree, which is equivalent to 0.0036 seconds, will be called a micro-degree. Geographic position computations will be performed to hundredths of a micro-degree, but the final positions will be rounded off to tenths of a micro-degree.

Azimuth is carried to hundredths of a second in first and second order work, and to tenths of a second in third order work. Computations will be carried to tenths of a micro-degree,

but the final result will be rounded off to the nearest micro-degree.

Equations A through H will be analyzed using limits of λ of ten degrees and the square-root-of-ten degrees, and of η corresponding to these values of λ.

TABLE I

Coefficients for the equation

$$\xi = \xi_r + A_2\lambda^2 + A_4\lambda^4 + A_6\lambda^6 + A_8\lambda^8$$

(expressed in meters for λ in degrees)

φ	$A_2 \times 10^2$	$A_4 \times 10^4$	$A_6 \times 10^6$	$A_8 \times 10^8$
0	00 000.000	000.000	0.000	0.000
10	16 608.394	+205.595	+2.453	+0.029
20	31 222.685	+344.436	+3.432	+0.030
30	42 085.047	+377.580	+2.721	+0.011
40	47 883.665	+308.979	+0.884	-0.007
45	48 636.583	+248.809	+0.119	-0.010
50	47 911.716	+181.153	-0.420	-0.009
60	42 156.076	+ 53.914	-0.715	-0.003
70	31 303.457	- 23.624	-0.355	+0.001
80	16 661.114	- 34.431	-0.030	0.000

φ	$A_2 \times 10$	$A_4 \times 10^2$	$A_6 \times 10^3$	$A_8 \times 10^4$
0	0 000.000	0.000	0.000	0.000
10	1 660.839	+2.056	+0.002	0.000
20	3 122.268	+3.444	+0.003	0.000
30	4 208.505	+3.776	+0.003	0.000
40	4 788.366	+3.090	+0.001	0.000
45	4 863.658	+2.488	0.000	0.000
50	4 791.172	+1.812	0.000	0.000
60	4 215.608	+0.539	-0.001	0.000
70	3 130.346	-0.236	0.000	0.000
80	1 666.111	-0.344	0.000	0.000

By examining Tables I, II and III, it is seen that the equations given are satisfactory for $\lambda = \pm 10^\circ$ at all latitudes, and that as the latitude increases, it would be possible to

TABLE II

Coefficients for the equation

$$\eta = B_1\lambda + B_3\lambda^3 + B_5\lambda^5 + B_7\lambda^7$$

(expressed in meters for λ in degrees)

φ	$B_1 \times 10$	$B_3 \times 10^3$	$B_5 \times 10^5$	$B_7 \times 10^7$
0	1 112 793.420	+5 687.842	+43.845	+0.381
10	1 095 998.680	+5 264.191	+36.046	+0.260
20	1 046 095.172	+4 096.475	+16.846	+0.003
30	964 518.229	+2 582.315	- 4.023	-0.188
40	853 635.589	+ 762.670	-16.559	-0.182
45	788 189.574	+ 67.709	-18.512	-0.133
50	716 704.929	- 627.646	-17.730	-0.074
60	557 804.713	-1 414.777	-10.856	+0.010
70	381 732.485	-1 484.445	- 0.300	+0.021
80	193 867.587	- 924.893	+ 0.627	+0.004
φ	$B_1 \times 10^{1/2}$	$B_3 \times 10^{1\frac{1}{2}}$	$B_5 \times 10^{2\frac{1}{2}}$	$B_7 \times 10^{3\frac{1}{2}}$
0	351 896.177	+179.865	+0.139	0.000
10	346 585.214	+166.468	+0.114	0.000
20	330 804.339	+129.542	+0.053	0.000
30	305 007.445	+ 81.660	-0.013	0.000
40	269 943.275	+ 24.118	-0.052	0.000
45	249 247.428	+ 2.141	-0.059	0.000
50	226 641.999	- 19.848	-0.056	0.000
60	176 393.338	- 44.739	-0.034	0.000
70	120 714.411	- 46.942	-0.001	0.000
80	61 306.314	- 29.248	+0.002	0.000

increase the value of λ to obtain a wider projection plane, without sacrificing any accuracy. In its cartographic application, the Universal Transverse Mercator Grid system is designed for zones with $\lambda = 3^\circ$, but with a small overlap at the zone boundaries for surveying and artillery purposes. For the sake of convenience, the amount of overlap considered here was taken as the difference between three and the square root of ten. The second part of the first three tables is designed so that the values in the body of the table are the values in

TABLE III

Coefficients for the equation

$$c = C_1\lambda + C_3\lambda^3 + C_5\lambda^5 + C_7\lambda^7$$

(expressed in micro-degrees for λ in degrees)

φ	$C_1 \times 10$	$C_3 \times 10^3$	$C_5 \times 10^5$	$C_7 \times 10^7$
0	0 000 000.0	00 000.0	000.0	0.0
10	1 736 481.8	17 438.6	+210.1	+2.3
20	3 420 201.4	31 218.0	+322.9	+2.9
30	5 000 000.0	40 465.2	+314.3	+1.7
40	6 427 876.1	38 758.5	+180.5	0.0
45	7 071 067.8	36 264.8	+109.7	-0.6
50	7 660 444.4	32 408.4	+ 45.6	-0.8
60	8 660 254.0	22 095.6	- 35.1	-0.5
70	9 396 926.2	11 188.0	- 44.8	-0.1
80	9 848 077.5	3 017.1	- 16.8	0.0

φ	$C_1 \times 10^{1/2}$	$C_3 \times 10^{1\frac{1}{2}}$	$C_5 \times 10^{2\frac{1}{2}}$	$C_7 \times 10^{3\frac{1}{2}}$
0	000 000.0	000.0	0.0	0.0
10	549 123.8	551.5	+0.7	0.0
20	1 081 562.6	987.2	+1.0	0.0
30	1 581 138.8	1 279.6	+1.0	0.0
40	2 032 672.9	1 225.7	+0.6	0.0
45	2 236 068.0	1 146.8	+0.3	0.0
50	2 422 445.2	1 024.8	+0.1	0.0
60	2 738 612.8	698.7	-0.1	0.0
70	2 971 569.0	353.8	-0.1	0.0
80	3 114 235.5	95.4	-0.1	0.0

meters for Tables I and II, and in micro-degrees for Table III, when λ is at this adopted Universal Transverse Mercator Grid system limit. In each case, the last term of the equation may be eliminated. Figure 3 shows the extent of the projection plane (on one side of the central meridian only) as a function of latitude. By using a value of $\lambda = 10^\circ$, it would be possible to depict the area from Fort Worth, Texas, to Los Angeles, California, on one plane. This is more than ample for most triangulation nets.

Projection Plane Width

as a function of latitude

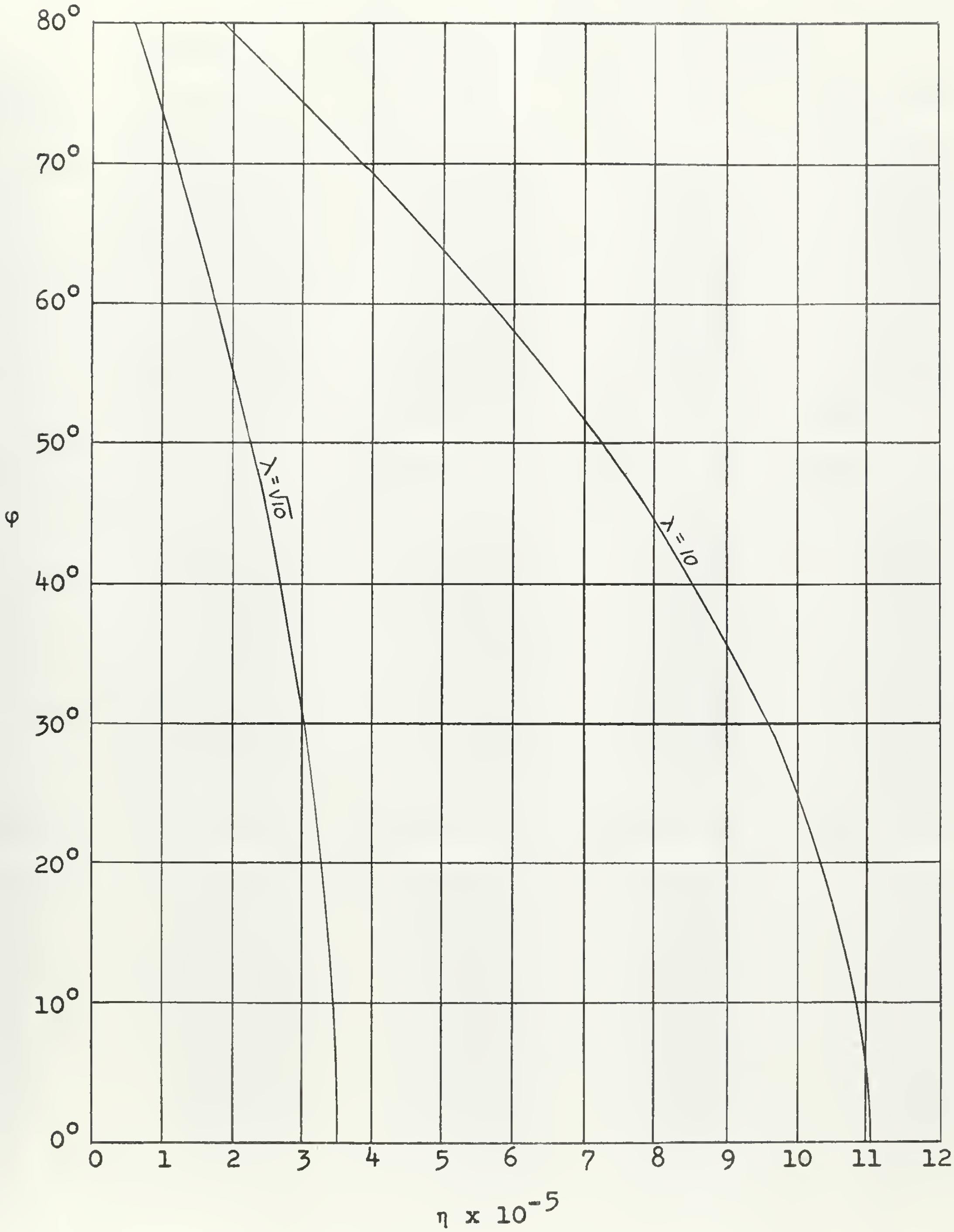

Fig. 3

TABLE IV

Coefficients for the equation

$$\varphi = \varphi_1 - D_2\eta^2 + D_4\eta^4 - D_6\eta^6 + D_8\eta^8$$

(expressed in milli-degrees for η in meters)

$\xi x10^{-6}$	$D_2 \times 10^{12}$	$D_4 \times 10^{24}$	$D_6 \times 10^{36}$	$D_8 \times 10^{48}$
0	0000.00000	0000.00000	000.00000	000.00000
1	112.92484	1.17600	0.01204	0.00012
2	231.43981	2.51796	0.02767	0.00031
3	362.39609	4.28769	0.05319	0.00071
4	515.78737	6.94480	0.10469	0.00179
5	708.47016	11.58640	0.23293	0.00563
6	973.16486	21.26604	0.64304	0.02440
7	1385.23020	46.99011	2.51304	0.17174
8	2169.61480	148.94059	18.18890	3.09524
9	4450.36547	1143.10813	569.22709	356.27858

$\xi x10^{-6}$	$D_2 x10^8/E_1^2$	$D_4 x10^{16}/E_1^4$	$D_6 x10^{24}/E_1^6$	$D_8 x10^{32}/E_1^8$
0	000.00000	0.00000	0.00000	0.00000
1	136.40139	1.71580	0.02122	0.00026
2	259.13397	3.15662	0.03884	0.00049
3	355.98854	4.13741	0.05042	0.00066
4	417.48436	4.54986	0.05552	0.00077
5	437.79482	4.42432	0.05496	0.00082
6	415.24151	3.87182	0.04996	0.00081
7	352.33909	3.04007	0.04135	0.00072
8	255.43295	2.06444	0.02968	0.00059
9	134.00879	1.03648	0.01554	0.00029

$\xi x10^{-6}$	$D_2 x10^7/E_1^2$	$D_4 x10^{14}/E_1^4$	$D_6 x10^{21}/E_1^6$	$D_8 x10^{28}/E_1^8$
0	00.00000	0.00000	0.00000	0.00000
1	13.64014	0.01716	0.00002	0.00000
2	25.91340	0.03157	0.00004	0.00000
3	35.59885	0.04137	0.00005	0.00000
4	41.74844	0.04550	0.00006	0.00000
5	43.77948	0.04424	0.00005	0.00000
6	41.52415	0.03872	0.00005	0.00000
7	35.23391	0.03040	0.00004	0.00000
8	25.54330	0.02064	0.00003	0.00000
9	13.40088	0.01036	0.00002	0.00000

TABLE V

Coefficients for the equation

$$\lambda = E_1\eta - E_3\eta^3 + E_5\eta^5 - E_7\eta^7$$

(expressed in milli-degrees for η in meters)

$\xi x10^{-6}$	$E_1 \times 10^6$	$E_3 \times 10^{18}$	$E_5 \times 10^{30}$	$E_7 \times 10^{42}$
0	8986.39390	37.09277	0.22842	0.00162
1	9098.82588	39.43536	0.26448	0.00211
2	9450.54504	47.22011	0.39514	0.00411
3	10089.59511	63.20573	0.71243	0.01005
4	11115.14797	94.17351	1.49587	0.02931
5	12721.12362	157.22636	3.68694	0.10496
6	15308.85909	301.95967	11.22391	0.50088
7	19828.07533	710.79751	47.29804	3.75708
8	29144.24802	2394.89065	355.28402	64.32891
9	57627.68868	19197.74878	11557.78945	8283.89718

$\xi x10^{-6}$	$E_1 x10^4/E_1$	$E_3 x10^{12}/E_1^3$	$E_5 x10^{20}/E_1^5$	$E_7 x10^{28}/E_1^7$
0	10000.00000	51.11319	0.38976	0.00342
1	10000.00000	52.35158	0.42409	0.00409
2	10000.00000	55.94438	0.52417	0.00610
3	10000.00000	61.53684	0.68136	0.00944
4	10000.00000	68.57772	0.88169	0.01398
5	10000.00000	76.37460	1.10672	0.01947
6	10000.00000	84.16286	1.33484	0.02542
7	10000.00000	91.18097	1.54326	0.03118
8	10000.00000	96.74469	1.68971	0.03602
9	10000.00000	100.31288	1.81853	0.03925

$\xi x10^{-6}$	$E_1 x10^{3\frac{1}{2}}/E_1$	$E_3 x10^{10\frac{1}{2}}/E_1^3$	$E_5 x10^{17\frac{1}{2}}/E_1^5$	$E_7 x10^{24\frac{1}{2}}/E_1^7$
0	3162.27766	1.61634	0.00123	0.00000
1	3162.27766	1.65550	0.00134	0.00000
2	3162.27766	1.76912	0.00166	0.00000
3	3162.27766	1.94597	0.00215	0.00000
4	3162.27766	2.16862	0.00279	0.00000
5	3162.27766	2.41518	0.00350	0.00001
6	3162.27766	2.66146	0.00422	0.00001
7	3162.27766	2.88340	0.00488	0.00001
8	3162.27766	3.05934	0.00534	0.00001
9	3162.27766	3.17217	0.00575	0.00001

TABLE VI

Coefficients for the equation

$$c = F_1\eta - F_3\eta^3 + F_5\eta^5 - F_7\eta^7$$

(expressed in micro-degrees for η in meters)

$\xi \times 10^{-6}$	$F_1 \times 10^6$	$F_3 \times 10^{18}$	$F_5 \times 10^{30}$	$F_7 \times 10^{42}$
0	0000000.0	00000.0	000.0	0.0
1	1430656.9	11948.1	123.6	1.3
2	2934256.6	26461.1	305.1	3.7
3	4599726.7	47341.8	653.3	9.8
4	6555921.3	82004.0	1456.4	29.2
5	9019124.3	147950.4	3660.4	104.9
6	12408164.3	295038.7	11206.1	500.8
7	17686980.3	705870.8	47286.4	3757.0
8	27733076.1	2391723.4	359632.6	64328.9
9	56927366.0	19196199.6	11557786.2	8283897.2

$\xi \times 10^{-6}$	$F_1 \times 10^4/E_1$	$F_3 \times 10^{12}/E_1^3$	$F_5 \times 10^{20}/E_1^5$	$F_7 \times 10^{28}/E_1^7$
0	0 000000.0	00000.0	000.0	0.0
1	1 572353.3	15861.4	198.2	2.5
2	3 104854.3	31350.0	404.7	5.4
3	4 558881.4	46091.8	624.8	9.2
4	5 898186.2	59715.8	858.4	13.9
5	7 089880.2	71868.7	1098.7	19.4
6	8 105218.2	82233.8	1332.7	25.4
7	8 920170.0	90549.0	1542.9	31.2
8	9 515797.4	96616.7	1710.4	36.0
9	9 878474.6	100304.8	1818.5	39.2

$\xi \times 10^{-6}$	$F_1 \times 10^{3\frac{1}{2}}/E_1$	$F_3 \times 10^{10\frac{1}{2}}/E_1^3$	$F_5 \times 10^{17\frac{1}{2}}/E_1^5$	$F_7 \times 10^{24\frac{1}{2}}/E_1^7$
0	000000.0	000.0	0.0	0.0
1	497221.8	501.6	0.6	0.0
2	981841.1	991.4	1.3	0.0
3	1 441644.9	1457.6	2.0	0.0
4	1 865170.2	1888.4	2.7	0.0
5	2 242017.0	2272.7	3.5	0.0
6	2 563095.0	2600.5	4.2	0.0
7	2 820805.4	2863.4	4.9	0.0
8	3 009159.4	3055.3	5.4	0.0
9	3 123848.0	3171.9	5.8	0.0

TABLE VII

Coefficients for the equation

$$m = m_o + G_2\lambda^2 + G_4\lambda^4 + G_6\lambda^6$$

(dimensionless for λ in degrees)

φ	$G_2 \times 10^2$	$G_4 \times 10^4$	$G_6 \times 10^6$
0	0.015327 82253	+0.000196 92397	+0.000002 39380
10	0.014862 61867	+0.000180 39090	+0.000002 01954
20	0.013524 16410	+0.000136 56597	+0.000001 12602
30	0.011476 54617	+0.000080 22432	+0.000000 22258
40	0.008969 75130	+0.000028 75541	-0.000000 27972
45	0.007638 15029	+0.000009 20031	-0.000000 34828
50	0.006308 10286	-0.000004 97203	-0.000000 32365
60	0.003812 63490	-0.000017 19540	-0.000000 14655
70	0.001782 37081	-0.000013 40684	+0.000000 00908
80	0.000459 17694	-0.000004 35159	+0.000000 00013

φ	$G_2 \times 10$	$G_4 \times 10^2$	$G_6 \times 10^3$
0	0.001532 78225	+0.000001 96924	+0.000000 00239
10	0.001486 26187	+0.000001 80391	+0.000000 00202
20	0.001352 41641	+0.000001 36566	+0.000000 00113
30	0.001147 65462	+0.000000 80224	+0.000000 00022
40	0.000896 97513	+0.000000 28755	-0.000000 00028
45	0.000763 81503	+0.000000 09200	-0.000000 00035
50	0.000630 81029	-0.000000 04972	-0.000000 00032
60	0.000381 26349	-0.000000 17195	-0.000000 00015
70	0.000178 23708	-0.000000 13407	+0.000000 00001
80	0.000045 91769	-0.000000 04352	0.000000 00000

It is more difficult to analyze the inverse equations, since they are functions of η, and the maximum η usable is a function of the latitude. For this reason, Tables IV, V and VI give values of the various terms for η = 1,000,000, and in addition give values of the terms for such values of η that $E_1\eta = 10^\circ$ for the middle part of the table, and $E_1\eta = 10^{1/2\,\circ}$ for the last part of the table. From these it is seen that the equations are satisfactory out to a value of $E_1\eta = 10^{1/2}$, without

TABLE VIII

Coefficients for the equation

$$m = m_o \, (1 + H_2\eta^2 + H_4\eta^4 + H_6\eta^6)$$

(dimensionless for η in meters)

$\xi x10^{-6}$	$H_2 \times 10^{12}$	$H_4 \times 10^{24}$	$H_6 \times 10^{36}$
0	0.012382 97846	0.000026 24824	0.000000 02068
1	0.012378 86260	0.000026 21302	0.000000 02067
2	0.012366 93350	0.000026 11131	0.000000 02064
3	0.012348 39964	0.000025 95428	0.000000 02059
4	0.012325 12548	0.000025 75881	0.000000 02053
5	0.012299 42966	0.000025 54523	0.000000 02047
6	0.012273 84290	0.000025 33485	0.000000 02040
7	0.012250 85498	0.000025 14780	0.000000 02035
8	0.012232 67694	0.000025 00119	0.000000 02030
9	0.012221 04007	0.000024 90793	0.000000 02027

using the last term. When $E_1\eta$ is increased to 10^o there is a loss of accuracy in the higher latitudes when converting from rectangular coordinates to longitude. This loss is caused primarily by the convergence of the meridians, a given arc at 80^o being only one sixth the lineal distance of the same arc when measured at the equator. This slight loss in angular accuracy would probably be acceptable under most conditions.

Tables VII and VIII give factors affecting the scale factor. If $\lambda^o = 10^{1/2}$ or less, the last term of Equation G may be omitted. The last term of Equation H may be neglected for any value of η which corresponds to $\lambda^o = 10$ or less, for an accuracy of four parts per hundred million or better.

Equation I is more complicated than those heretofore encountered, as it is variable with ξ, η, difference in ξ and

TABLE IX

Factors affecting the (T-t) correction

Coefficients for Equation I in milli-degrees for η in meters

$\xi x10^{-6}$	$I_1 \times 10^{10}$	$I_2 \times 10^{20}$	$I_3 \times 10^{25}$	$I_4 \times 10^{35}$
0	7.0949 24040	00.000 00000	57.783 35594	000.000
1	7.0925 65824	23.381 77552	57.764 27909	96.480
2	7.0857 30951	44.388 35104	57.708 96956	182.983
3	7.0751 11839	60.910 50009	57.622 98212	250.716
4	7.0617 76719	71.331 58182	57.514 90771	293.057
5	7.0470 54102	74.684 84513	57.395 46662	306.194
6	7.0323 93969	70.726 85122	57.276 40567	289.363
7	7.0192 22862	59.928 54990	57.169 33058	244.725
8	7.0088 07609	43.397 64799	57.084 58741	176.956
9	7.0021 40178	22.751 63119	57.030 30507	92.683

Correction (milli-degrees) per 10^5 change in η or ξ, at $\eta_3 = 10^6$

$\xi x10^{-6}$	$(T-t)_1$	$(T-t)_2$	$(T-t)_3$	$(T-t)_4$
0	70.9492	0.0000	0.5778	0.0000
1	70.9257	0.0234	0.5776	0.0001
2	70.8573	0.0444	0.5771	0.0002
3	70.7511	0.0609	0.5762	0.0003
4	70.6178	0.0713	0.5751	0.0003
5	70.4705	0.0747	0.5740	0.0003
6	70.3239	0.0707	0.5728	0.0003
7	70.1922	0.0599	0.5717	0.0002
8	70.0881	0.0434	0.5708	0.0002
9	70.0214	0.0228	0.5703	0.0001

Correction (milli-degrees) per 10^5 change in η or ξ, at $\eta_3 = 10^{5\frac{1}{2}}$

$\xi x10^{-6}$	$(T-t)_1$	$(T-t)_2$	$(T-t)_3$	$(T-t)_4$
0	22.4361	0.0000	0.0183	0.0000
1	22.4287	0.0023	0.0183	0.0000
2	22.4070	0.0044	0.0182	0.0000
3	22.3735	0.0061	0.0182	0.0000
4	22.3313	0.0071	0.0182	0.0000
5	22.2847	0.0074	0.0181	0.0000
6	22.2384	0.0071	0.0181	0.0000
7	22.1967	0.0060	0.0181	0.0000
8	22.1638	0.0043	0.0181	0.0000
9	22.1427	0.0023	0.0181	0.0000

TABLE X

Coefficients for the equation

$$S = \frac{s}{m_o}\left(1 - J_2\frac{\eta_2^3 - \eta_1^3}{\eta_2 - \eta_1} + J_4\frac{\eta_2^5 - \eta_1^5}{\eta_2 - \eta_1} - J_6\frac{\eta_2^7 - \eta_1^7}{\eta_2 - \eta_1}\right)$$

(dimensionless for η in meters)

$\xi \times 10^{-6}$	$3J_2 \times 10^{12}$	$5J_4 \times 10^{24}$	$7J_6 \times 10^{36}$
0	0.0123 82978	0.0001 27090	0.0000 01269
1	0.0123 78863	0.0001 27023	0.0000 01269
2	0.0123 66933	0.0001 26830	0.0000 01266
3	0.0123 48400	0.0001 26529	0.0000 01263
4	0.0123 25125	0.0001 26150	0.0000 01258
5	0.0122 99430	0.0001 25731	0.0000 01253
6	0.0122 73843	0.0001 25312	0.0000 01248
7	0.0122 50855	0.0001 24936	0.0000 01243
8	0.0122 32677	0.0001 24637	0.0000 01239
9	0.0122 21040	0.0001 24446	0.0000 01237

difference in η. The top third of Table IX gives the values of the coefficients I as a function of ξ, the middle third gives the magnitude of the individual terms of the correction, in milli-degrees, for a line 100,000 meters in ξ and 100,000 meters in η and with η_3 = 1,000,000 meters, while the bottom third gives the magnitude of the individual terms of the correction for a line of the same length, but with η_3 = 316,000 meters. Since there are so many variables involved, it is not possible to establish general rules about the necessity of including the various terms, but a decision will have to be made in this respect for each different line considered.

Equation J has been developed along lines suggested by Bomford (1). Bomford gives the equation to the J_2 term, then cites an unpublished paper of A. R. Robbins, which gives the

TABLE XI

Comparison of finite distance scale factors

obtained from Equations J and K

	$\xi_1 = 5.0 \times 10^6$, $\xi_2 = 5.0 \times 10^6$; $\eta_1 = 0.5 \times 10^6$, $\eta_2 = 1.0 \times 10^6$	$\xi_1 = 4.5 \times 10^6$, $\xi_2 = 5.5 \times 10^6$; $\eta_1 = 0.5 \times 10^6$, $\eta_2 = 1.0 \times 10^6$
For one interval		
L_K	= 0.9928 73761	= 0.9928 75348
L_J	= 0.9928 73698	= 0.9928 73698
Diff.	= 0.0000 00063	= 0.0000 01650
For five intervals		
L_K	= 0.9928 73701	= 0.9928 75292
L_J	= 0.9928 73698	= 0.9928 75225
Diff.	= 0.0000 00003	= 0.0000 00067
	$\xi_1 = 5.5 \times 10^6$, $\xi_2 = 4.5 \times 10^6$; $\eta_1 = 0.5 \times 10^6$, $\eta_2 = 1.0 \times 10^6$	$\xi_1 = -0.5 \times 10^6$, $\xi_2 = +0.5 \times 10^6$; $\eta_1 = 1.0 \times 10^6$, $\eta_2 = 1.0 \times 10^6$
For one interval		
L_K	= 0.9928 72168	= 0.9877 43194
L_J	= 0.9928 73698	= 0.9877 42843
Diff.	=-0.0000 01530	= 0.0000 00351
For five intervals		
L_K	= 0.9928 72104	= 0.9877 43194
L_J	= 0.9928 72166	= 0.9877 43169
Diff.	=-0.0000 00062	= 0.0000 00025

equation developed through the J_4 term, expressing it logarithmically. Tardi and Laclavere (5) give Equation J only through the J_2 term, then give Equation K and state that it is more accurate. Equation J has the drawback of having assumed a value of the radius of curvature in the prime vertical at the footpoint of the midpoint of the line, while

Equation K takes into account the change of radius of curvature with change of latitude, and performs a mechanical integration over the extent of the line in accordance with Simpson's rule.

In comparing Equation J with Equation K, the equations were put into the form ——

$$S = \frac{s}{m_o} L ,$$

and the factor L was evaluated in each case. It should be kept in mind that the values given in Table X produce an error of twelve parts per billion caused by the elimination of the J_8 term. All of this error is caused by the effect of the H_2 and H_4 terms in the expansion of the reciprocal of the scale factor.

Table XI gives the factor L for four different lines considered. The line of constant η was originally tried at $\xi = 5 \times 10^6$, where the rate of change of the radius of curvature is a maximum. The agreement between the two equations was very good, because the rate of change is almost constant, even though maximum. In order to get the effect of the maximum value of the second derivative of the radius of curvature the line was moved to the vicinity of the equator. The lines tested were, of course, extreme in length, but they show a definite tendency on the part of Equation J to give inaccurate answers since no provision is made for the change of radius of curvature along the line.

The values given in the preceding tables were computed using the parameters of the International Ellipsoid.

The Army Map Service has tabulated the following factors for the International, Clarke 1866, Clarke 1880, Bessel and Everest (to 45° only) Ellipsoids in a series of Technical Manuals —

AMS title	Designation in this paper, or function if no specific designation is available
I	ξ_r, argument φ,
II	A_2, argument φ,
III	A_4, argument φ,
A_6	$\frac{N \sin \varphi}{720} \cos^5\varphi \; m_o \; (61 - 58z^2 + z^4 + 270n^2 - 330n^2z^2)\lambda^6$, arguments φ, λ,
IV	B_1, argument φ,
V	B_3, argument φ,
B_5	$\frac{N}{120} \cos^5\varphi \; m_o \; (5 - 18z^2 + z^4 + 14n^2 - 58n^2z^2)\lambda^5$, arguments φ, λ,
VII	D_2, argument φ_1,
VIII	$\frac{z_1}{24 \; m_o^4 N_1^4} \; (5 + 3z_1^2 + 6n_1^2 - 6n_1^2z_1^2 - 3n_1^4 - 9n_1^2\sin^2\varphi_1)$, argument φ_1,
D_6	$\frac{z_1}{720 \; m_o^6 N_1^6} (61 + 90z_1^2 + 45z_1^4 + 107n_1^2 - 162n_1^2z_1^2 - 90n_1^2z_1^4) \; \eta^6$, arguments φ_1, η,
IX	E_1, argument φ_1,
X	E_3, argument φ_1,
E_5	$\frac{1}{120 \; m_o^5 N_1^5 \cos \varphi_1} (5 + 28z_1^2 + 24z_1^4 + 6n_1^2 + 8n_1^2z_1^2) \; \eta^5$, arguments η, φ_1
XII	C_1, argument φ,

AMS title	Designation in this paper, or function if no specific designation is available
XIII	C_3, argument φ,
C_5	$\frac{1}{15} z \cos^5\varphi\ (2 - z^2)\lambda^5$, arguments φ, λ,
XV	F_1, argument φ_1
XVI	F_3, argument φ_1
F_5	$\frac{z_1}{15 m_o^5 N_1^5}\ (2 + 5z_1^2 + 3z_1^4)\ \eta^5$, arguments φ_1, η, and
XVIII	H_2, argument ξ.

φ_1 is obtained by inverse interpolation in AMS function I, and is then used as an argument for finding other factors.

The Army Map Service tables are sufficient for use within the standard Universal Transverse Mercator Grid system zones, but if the zone is extended, factors including additional terms must be computed.

This paper makes no attempt to present the factors necessary for the computation of Equations A through G. The transformation of coordinates, although important, is adequately dealt with in such publications as the Army Map Service Manuals, and the extension of the tables for zones wider than 6^o can be performed by the individual desiring the information.

CHAPTER IV

APPLICATIONS

In order to compare the results of computation on a plane with ordinary ellipsoidal computations, a net given by Reynolds (4), which he adjusted by the direction method, will be computed on the plane and adjusted by the variation of coordinates method.

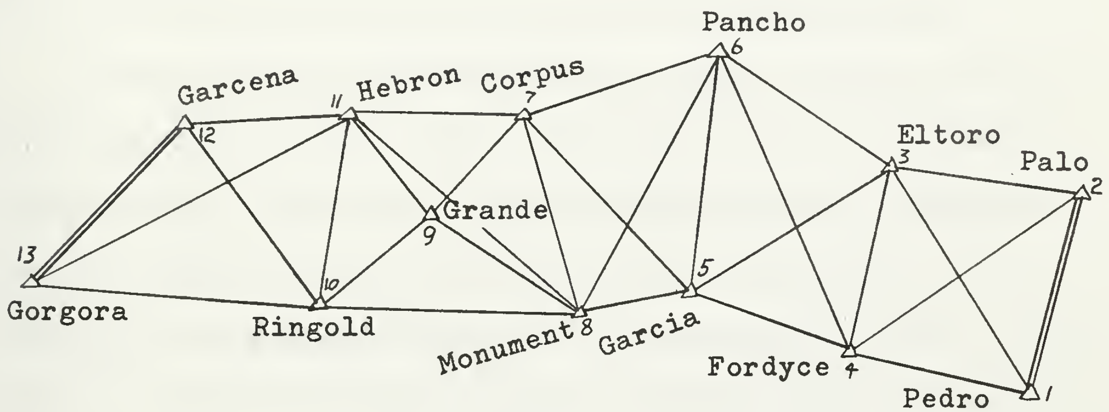

Fig. 4

In this net, stations Garcena, Gorgora, Palo and Pedro are fixed in position, the fixed positions requiring that the lines Garcena-Gorgora and Palo-Pedro remain fixed in length and azimuth.

Palo	$\varphi = 26^\circ.327\ 4864$ N,	$\lambda = 98^\circ.463\ 4022$ W
Pedro	$\varphi = 26.243\ 5389$ N,	$\lambda = 98.483\ 2561$ W
Garcena	$\varphi = 26.448\ 9847$ N,	$\lambda = 98.928\ 8656$ W
Gorgora	$\varphi = 26.423\ 2164$ N,	$\lambda = 99.009\ 8733$ W

α, Garcena-Gorgora = $70^\circ.559\ 036$, α, Palo-Pedro = $12^\circ.040\ 278$

α, Gorgora-Garcena = 250.522 972, α, Pedro-Palo = 192.031 486

S, Gorgora-Garcena = 8,569.81 m., S, Pedro-Palo = 9,509.38 m.

In the Universal Transverse Mercator Grid system, this net would fall in zone 14, with central meridian of 99°W, which would put all the stations within six tenths of a degree of the central meridian, making many of the (T-t) components negligible. For the sake of illustration, a non-standard zone will be used, with central meridian (λ_o) at 108°W, putting the net between 9 and 10 degrees from the central meridian.

Quite frequently preliminary positions are used for an extended period of time before the final adjustment is made, generally for some type of graphical plotting such as hydrographic survey control or photogrammetric compilation. The accuracy of the position used is not important in these cases, as a very large error must exist to become apparent through these graphical methods (at the most commonly used scales). The question of internal consistency is of far greater importance than that of the position used, since the computation of non-unique answers to a problem tends to confuse the survey personnel and waste time by their trying to find non-existent computational errors. In order to make each figure discrepancy free within itself, an engineers adjustment is included in the computation of the preliminary positions.

Computations for the preliminary positions will be made on a form designed to facilitate the following functions ——

1. Using T (grid azimuth of the geodesic) values, a rough approximation of the coordinates of the new stations is obtained by use of intersection formulas,

2. Using these rough coordinates, (T-t) corrections are computed for enough lines to locate the new stations more accurately, then more accurate (T-t) corrections are computed for each line in the figure,

3. An engineers adjustment is performed on the plane angles to achieve internal consistency, and

4. Preliminary positions are computed by intersection formulas, working around the figure until a check is obtained on a previously known station.

Equation I is a function of φ, which is not explicitly known, and some variables which are known. φ can be computed from Equation D, but this is an undesirable addition to the required work. Since ξ_r is a function of φ, if ξ can be reduced to ξ_r, this value could then be used as an argument for determining the various coefficients of Equation I. The coefficients I are slowly-changing functions of φ, so a rigorous correspondence between ξ and ξ_r is not required. Assuming a sphere of reasonable radius, $m_o r$, as in Fig. 5, by the cosine law we have

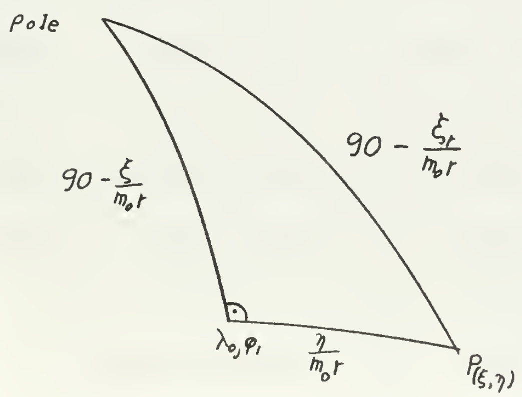

Fig. 5

$$\cos\left(90 - \frac{\xi_r}{m_o r}\right) = \cos\left(90 - \frac{\xi}{m_o r}\right) \times \cos\frac{\eta}{m_o r}\text{, or}$$

$$\sin\frac{\xi_r}{m_o r} = \sin\frac{\xi}{m_o r} \times \cos\frac{\eta}{m_o r}\text{,}$$

which may be expanded into series, as

$$\frac{\xi_r}{m_o r} - \frac{\xi_r^3}{3!m_o^3 r^3} + \frac{\xi_r^5}{5!m_o^5 r^5} - \dots = \left(\frac{\xi}{m_o r} - \frac{\xi^3}{3!m_o^3 r^3} + \frac{\xi^5}{5!m_o^5 r^5} - \dots\right) \times$$

$$\left(1 - \frac{\eta^2}{2!m_o^2 r^2} + \frac{\eta^4}{4!m_o^4 r^4} - \dots\right),$$

$$\xi_r - \frac{\xi_r^3}{3!m_o^2 r^2} + \frac{\xi_r^5}{5!m_o^4 r^4} - \dots = \xi - \frac{\xi\eta^2}{2!m_o^2 r^2} - \frac{\xi^3}{3!m_o^2 r^2} + \dots ,$$

$$\xi_r \doteq \xi\left(1 - \frac{\eta^2}{2m_o^2 r^2}\right) = \xi\,(1 - H_2\eta^2).$$

As a starting point for the calculations it is necessary to find the convergence of the meridians at the four known points, and to find the correction between the projected geodesic and the rectilinear chord between the known points.

	Palo-Pedro	Pedro-Palo	Garcena-Gorgora	Gorgora-Garcena
α	12°040 278	192°031 486	70°559 036	250°522 972
c	4.261 641	4.240 085	4.067 995	4.027 542
T	187.778 637	7.791 401	246.491 041	66.495 430

Gorgora-Garcena ξ_3 = 2955 086.12; ξ_{3r} = 2925 434; η_3 = 901 363.40

Garcena-Gorgora ξ_3 = 2956 236.52; ξ_{3r} = 2926 398; η_3 = 904 008.28

Pedro-Palo ξ_3 = 2940 882.19; ξ_{3r} = 2907 854; η_3 = 953 575.55

Palo-Pedro ξ_3 = 2944 056.86; ξ_{3r} = 2910 964; η_3 = 954 009.58

Line	Palo-Pedro	Pedro-Palo	Garcena-Gorgora	Gorgora-Garcena
$\xi_2-\xi_1$	-9,524.01	+9,524.01	-3,451.20	+3,451.20
$\eta_2-\eta_1$	-1,302.09	+1,302.09	-7,934.64	+7,934.64
$(T-t)_1$	-.006 4300	+.006 4271	-.002 2079	+.002 2014
$(T-t)_2$	-.000 0007	+.000 0007	-.000 0039	+.000 0039
$(T-t)_3$	+.000 0477	-.000 0476	+.000 0147	-.000 0146
$(T-t)_4$	negligible	negligible	negligible	negligible
(T-t)	-.006 3830	+.006 3802	-.002 1971	+.002 1907
T	187.778 637	7.791 401	246.491 041	66.495 430
t	187.785 020	7.785 021	246.493 238	66.493 239

The one micro-degree difference between forward and backward grid directions is not considered significant since the data were originally listed to the nearest hundredth of a second, which corresponds to 2.8 micro-degrees.

Computation of the preliminary positions and the adjustment of the net are given in Appendix I.

After preliminary positions had been determined, the net was adjusted by the method of variation of coordinates. For this method, error equations are written in the form ——

$$v_{ij} = - dz_i + a_{ij}d\eta_j + b_{ij}d\xi_j + a_{ji}d\eta_i + b_{ji}d\xi_i + l_{ij},$$

where

$$a_{ij} = -a_{ji} = \rho(\xi_j-\xi_i) / s^2,$$

$$b_{ij} = -b_{ji} = -\rho(\eta_j-\eta_i) / s^2,$$

$l_{ij} = \bar{t}_{ij} - (t_{oij} + \bar{z}_i)$, and, for convenience,

$$\bar{z}_i = \left(\left[\bar{t}_i\right] - \left[t_{oi}\right] \right) / n.$$

It should be noted that z and n are here used in their usual sense for adjustments, as the constant station correction and the number of observations, respectively, and not as a shorthand notation as was the case in Chapters II and III.

In this method of adjustment, the number of unknowns is equal to three times the number of new stations plus the number of old stations, but of these only two times the number of new stations need be solved for, as the others are the orientation corrections (dz's) for which the numerical values are not explicitly required, and which are removed from the error equations by the use of Schreiber's first theorem.

The reasons for using this method of adjustment are ——

1. The number of unknowns for which solutions are obtained is quite frequently less than in other methods (in the sample net there were 18 normal equations as opposed to 27 by the direction correction method),

2. The standard error of the position of each station in each component can be readily obtained,

3. The tedious use of tables required for the formation of side and length equations is eliminated, and

4. After computation, the unknowns are simply added to the preliminary positions for the final answers, eliminating recomputation of triangles and positions, but requiring computation of tangent t and t if required.

The adjustment was carried out twice, using first the l_1 and sum_1 values shown, and secondly the l_2 and sum_2 values. The

TABLE XII

Geographic Positions

Station	Latitude			Corr.*	Longitude			Corr.*
Fordyce	26°	17'	47".434	0	98°	34'	45".239	-0".001
Eltoro	26	21	51.958	0	98	34	00.307	-0.002
Garcia	26	20	41.270	0	98	42	29.281	-0.002
Pancho	26	26	36.792	0	98	41	17.287	-0.002
Monument	26	21	16.682	0	98	46	02.967	-0.002
Corpus	26	26	28.446	0	98	45	56.996	-0.002
Grande	26	23	30.225	0	98	49	31.291	-0.001
Hebron	26	27	00.537	0	98	53	03.822	-0.001
Ringold	26	22	30.754	0	98	53	30.365	-0.001
Garcena	26	26	56.345	0	98	55	43.916	0
Gorgora	26	25	23.579	0	99	00	35.544	0

* When correction is applied, values will agree with those reported by Reynolds (4).

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first set was obtained by using, for $\bar{t}$, the values used in the preliminary position computation. The result of this adjustment was that the values of latitude obtained agreed with the values obtained by Reynolds in his adjustment on the ellipsoid, but the longitudes disagreed slightly. Since the preliminary positions were rounded to the nearest centimeter, the values of $\bar{t}$ between the preliminary positions might vary greatly from the values used in computing the positions, especially when short sides are involved. The l_2 values were computed using the $\bar{t}$ values

TABLE XIII

Azimuths

Station	Azimuth			Corr.*	Back Azimuth			Corr.*	To
Fordyce	253°	26'	51".76	+".17	73°	29'	56".60	+".17	Palo
	301	27	1.66	+.20	121	29	34.59	+.20	Pedro
Eltoro	291	37	4.73	+.14	111	39	49.85	+.13	Palo
	328	04	40.83	+.08	148	06	54.03	+.07	Pedro
	9	24	18.89	-.17	189	23	58.96	-.17	Fordyce
Garcia	261	12	19.79	+.27	81	16	5.74	+.27	Eltoro
	292	32	32.92	+.10	112	35	58.68	+.09	Fordyce
Pancho	305	52	9.90	+.41	125	55	24.22	+.41	Eltoro
	326	15	56.22	+.18	146	18	50.36	+.17	Fordyce
	10	20	26.71	+.09	190	19	54.71	+.08	Garcia
Monument	218	46	22.49	+.16	38	48	29.51	+.16	Pancho
	280	24	31.05	+.24	100	26	5.90	+.23	Garcia
Corpus	268	05	3.85	+.31	88	07	8.41	+.30	Pancho
	331	40	6.59	+.26	151	41	38.93	+.25	Garcia
	0	59	19.05	+.26	180	59	16.40	+.25	Monument
Grande	227	15	43.06	+.34	47	17	18.41	+.33	Corpus
	305	25	23.17	+.24	125	26	55.64	+.25	Monument
Hebron	274	44	51.18	+.21	94	48	01.27	+.21	Corpus
	312	11	20.76	+.11	132	14	27.91	+.10	Monument
	317	41	11.46	-.01	137	42	46.03	-.01	Grande
Ringold	185	03	40.99	+.06	5	03	52.79	+.07	Hebron
	254	32	48.39	-.01	74	34	34.63	-.01	Grande
	280	23	8.92	+.08	100	26	27.61	+.07	Monument
Garcena	268	19	26.89	+.37	88	20	38.20	+.37	Hebron
	335	37	47.13	+.22	155	38	46.53	+.23	Ringold
Gorgora	250	31	22.54	+.16	70	33	32.38	+.15	Garcena
	256	33	46.14	+.17	76	37	7.26	+.16	Hebron
	294	15	53.59	-.05	114	19	2.63	-.05	Ringold

* When correction is applied, values will agree with those reported by Reynolds (4).

actually existing between the preliminary positions, and the corrections to the positions were recomputed. Had the preliminary positions been carried to a number of places commensurate with angles listed to the nearest micro-degree and with the distance involved, the values of $\bar{t}$ taken from the preliminary position computation would have been satisfactory. The change between the first and second adjustments was very slight, and gave the results shown in Tables XII and XIII. Of a total of 29 lines involved, 14 were of slightly different lengths from the ones computed by Reynolds, with a relative accuracy of ——

Fordyce - Palo	1: 470 000,	Hebron - Corpus	1: 440 000,
Fordyce - Pedro	1: 490 000,	Hebron - Monument	1: 440 000,
Eltoro - Palo	1: 220 000,	Ringold - Monument	1: 480 000,
Eltoro - Pedro	1: 440 000,	Garcena - Hebron	1: 170 000,
Eltoro - Fordyce	1: 1 730 000,	Garcena - Ringold	1: 810 000,
Grande - Corpus	1: 410 000,	Gorgora - Hebron	1: 490 000,
Grande - Monument	1: 320 000,	Gorgora - Ringold	1: 530 000.

Those lines not listed agree in difference of latitude and difference of longitude with the values obtained by Reynolds. The above ratios are not in themselves accurate, since the positions were first rounded to the nearest centimeter on the plane, this rounded value was converted to geographic coordinates, rounded to the nearest tenth of a micro-degree, then changed to the sexagesimal system, and rounded to the nearest thousandth of a second for comparison with the values obtained by Reynolds. The order of magnitude of the ratios is indicative of the fact that

the computations on the plane, even out to $\lambda = 10^{\circ}$, are satisfactory for all orders of geodetic computations. The azimuth differences are less than those which could be caused by possible rounding-off errors.

The standard error of an observation was computed to be $\pm$ 125 micro-degrees, giving a probable error of $\pm$ 0".30, as opposed to a value of $\pm$ 0".32 obtained by Reynolds.

A time study was made between the computational method described herein, and the conventional method of logarithmic computation on the ellipsoid. Since the corrections and functions for the method on the plane were obtained from tables requiring extensive interpolation, the same type of tables were used in obtaining the functions required for conventional computation, in an attempt to make the comparison realistic. If extensive tables were available for both methods of computation, so that linear interpolation would be sufficient, a considerable saving of time would be effected in both methods. In the plane method, two approximations of the positions were made, the T-t corrections computed, an engineering adjustment made, and the 'final preliminary' positions computed. In the conventional method, the triangles were computed approximately in order to obtain the spherical excess, an engineering adjustment made, then the triangles recomputed, and preliminary positions determined. On the plane the computations took two hours thirty-one minutes. By the conventional method the computations took

three hours thirty-nine minutes, an increase of forty-five per cent. The saving of time in the adjustment procedure is obvious when the difference in the number of normal equations is considered. Similarly, the difference in time to compute an inverse position can readily be appreciated by anyone who has been confounded by the conventional method of computation, since

$$s = ((\Delta\xi)^2 + (\Delta\eta)^2)^{1/2}, \text{ and}$$

$$t = \text{arc tan } \Delta\eta/\Delta\xi.$$

CHAPTER V

CONCLUSIONS

Computation of geodetic positions on a Transverse Mercator projection plane is considered feasible in those cases where the extent of the network does not exceed 20° of longitude. It is especially economical for those agencies which presently do mapping on the Universal Transverse Mercator Grid system, or which can conveniently convert to this system. The state plane coordinate grid systems are adequate for the areas they cover, but are limited in extent.

A natural result of the adoption of this system of computation would be the preparation of more extensive tables to enable linear interpolation. These tables could be obtained easily by interpolation in the tables given in Appendix II, or could be computed by the equations given in Chapter II.

Computation of positions on a plane is considered easier to comprehend than computation on the ellipsoid and should tend to eliminate the confusion experienced by those who compute by rote.

The time saving of thirty per cent over the conventional method of computation should entice those who compute preliminary positions to desire the method on the plane, especially where the computations are made in the field and other phases of operations are delayed until these computations can be completed.

APPENDIX I

TRIANGULATION COMPUTATION

TABLE XIV

Observational data

At Palo

Pedro	0°.000 000
Fordyce	61.458 758
Eltoro	99.623 625

At Pedro

Fordyce	0°.000 000
Eltoro	26.622 169
Palo	70.538 594

At Fordyce

Garcia	0°.000 000
Pancho	33.714 261
Eltoro	76.799 908
Palo	140.848 200

At Eltoro

Palo	0.000 000
Pedro	36.460 119
Fordyce	77.787 383
Garcia	149.650 419
Pancho	194.305 672

At Garcia

Monument	0.000 000
Corpus	51.259 306
Pancho	89.897 022
Eltoro	160.770 644
Fordyce	192.107 703

At Pancho

Eltoro	0.000 000
Fordyce	20.396 331
Garcia	64.471 283
Monument	92.938 844
Corpus	142.249 786

At Monument

Ringold	0.000 000
Grande	25.007 564
Hebron	31.799 875
Corpus	80.546 783
Pancho	118.331 944
Garcia	179.967 700

At Corpus

Pancho	0.000 000
Garcia	63.584 364
Monument	92.904 419
Grande	139.204 178
Hebron	186.716 244

At Grande

Ringold	0.000 000
Hebron	63.136 475
Corpus	152.685 811
Monument	230.846 958

At Hebron

Corpus	0.000 000
Monument	37.441 819
Grande	42.939 022
Ringold	90.317 314
Gorgora	161.871 189
Garcena	173.596 636

At Garcena

Hebron	0.000 000
Ringold	67.305 608
Gorgora	162.234 906

At Ringold

Gorgora	0.000 000
Garcena	41.328 886
Hebron	70.744 011
Grande	140.229 353
Monument	166.068 583

At Fordyce

Garcia	0.000 000
Pedro	188.850 961

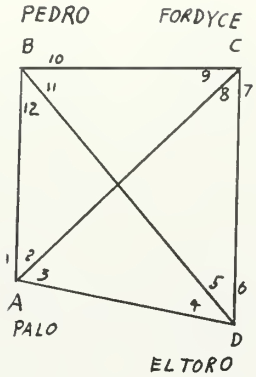
At Gorgora

Garcena	0.000 000
Hebron	6.040 025
Ringold	43.742 242

Positions

	Palo	Pedro	Garcena	Gorgora
ξ =	2947 231.53	2937 707.52	2957 386.92	2953 935.72
η =	954 443.61	953 141.52	906 653.16	898 718.52

ξ_B	2937 707.52
ξ_A	2947 231.53
$\xi_B - \xi_A$	- 9 524.01
$\overline{T}_{AB}$	187.778 637
-1 + 2	61.458 758
T'_{AC}	249.237 395
-2 + 3	38.164 867
T'_{AD}	287.402 262



η_B	953 141.52
η_A	954 443.61
$\eta_B - \eta_A$	- 1 302.09
$\overline{T}_{BA}$	7.791 400
+11 - 12	-43.916 425
T'_{BD}	323.874 975
+10 - 11	-26.622 169
T'_{BC}	297.252 806

- -

tan T'_{AC}	+2.6377 03229	$(T-t)_{AB}$	-0.006 383	tan T'_{AD}	-3.1905 62458
tan T'_{BC}	-1.9413 86401	$(T-t)_{BA}$	+0.006 380	tan T'_{BD}	-0.7298 81776
diff	+4.5790 89630			diff	-2.4606 80682

- -

$$\xi_i' - \xi_A = \frac{(\eta_B - \eta_A) - \tan T_{Bi}'(\xi_B - \xi_A)}{\tan T_{Ai}' - \tan T_{Bi}'}; \quad \eta_i' = \eta_A + \tan T_{Ai}' \ (\xi_i' - \xi_A).$$

$\xi_C' - \xi_A =$ -4 322.23; $\xi_C' =$ 2942 909.30; $\eta_C' =$ 943 042.85.

$\xi_D' - \xi_A =$ +3 354.15; $\xi_D' =$ 2950 585.68; $\eta_D' =$ 943 741.98.

$\xi_r' = \xi'(1 - H_2\eta^2)$ $\xi_{rA}' =$ 2914 073 $\xi_{rC}' =$ 2910 585

$\xi_{rB}' =$ 2904 745 $\xi_{rD}' =$ 2918 130

$$\xi_{3ij} = \frac{1}{3}(2\xi_{ri}' + \xi_{rj}'); \qquad \eta_{3ij} = \frac{1}{3}(2\eta'_i + \eta'_j)$$

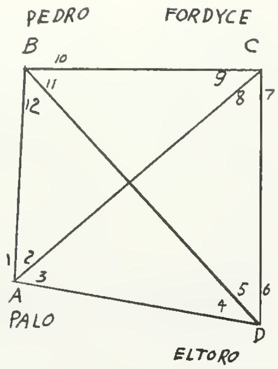
	ξ_3	η_3			ξ_3	η_3
AC	2912 910	950 643.36	;	BC	2906 692	949 775.30
AD	2915 425	950 876.40	;	BD	2909 207	950 008.34

- -

$$(T-t) = I_1(\xi_j - \xi_i)\eta_3 + I_2(\eta_j - \eta_i)\eta_3^2 - I_3(\xi_j - \xi_i)\eta_3^3 - I_4(\eta_j - \eta_i)\eta_3^4$$

Line	AC	AD	BC	BD
$\xi_j - \xi_i$	- 4 322.23	+ 3 354.15	+ 5 201.78	+12 878.16
$\eta_j - \eta_i$	-11 400.76	-10 701.63	-10 098.67	- 9 399.54
$(T-t)_1$	-.002 9078	+.002 2571	+.003 4963	+.008 6580
$(T-t)_2$	- 62	- 58	- 55	- 51
$(T-t)_3$	+ 214	- 166	- 257	- 636
$(T-t)_4$	negligible	negligible	negligible	negligible
$(T-t)$	-.002 8926	+.002 2347	+.003 4651	+.008 5893

ξ_B	2937 707.52	η_B	953 141.52
ξ_A	<u>2947 231.53</u>	η_A	<u>954 443.61</u>
$\xi_B - \xi_A$	− 9 524.01	$\eta_B - \eta_A$	− 1 302.09
$\bar{t}_{AB}$	187.785 020	$\bar{t}_{BA}$	7.785 020
−1' + 2'	<u>61.455 268</u>	+11' − 12'	<u>−43.918 634</u>
t''_{AC}	249.240 288	t''_{BD}	323.866 386
−2' + 3'	<u>38.159 740</u>	+10' − 11'	<u>−26.617 045</u>
t''_{AD}	287.400 028	t''_{BC}	297.249 341



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tan t''_{AC}	+2.6381 05075	$(T-t)_{AB}$	−.006 3830	tan t''_{AD}	−3.1909 98416
tan t''_{BC}	<u>−1.9416 74842</u>	$(T-t)_{BA}$	+.006 3802	tan t''_{BD}	<u>−0.7301 11567</u>
diff.	+4.5797 79917			diff.	−2.4608 86849

- -

$-1' + 2' = -1 + 2 + (T-t)_{AB} - (T-t)_{AC}$ etc.

- -

$\xi''_C - \xi_A =$ −4 322.18; $\xi''_C =$ 2942 909.35; $\eta''_C =$ 943 041.25.

$\xi''_D - \xi_A =$ +3 354.76; $\xi''_D =$ 2950 586.29; $\eta''_D =$ 943 738.58

$\xi''_r = \xi''(1 - H_2\eta^2)$ $\quad \xi''_{rA} =$ 2914 073 $\quad \xi''_{rC} =$ 2910 585

$\qquad \xi''_{rB} =$ 2904 745 $\quad \xi''_{rD} =$ 2918 130

	ξ_3	η_3		ξ_3	η_3
AC	2912 910	950 642.82	; CA	2911 748	946 841.89
AD	2915 425	950 874.97	; DA	2916 778	947 306.92
BC	2906 692	949 774.76	; CB	2908 638	946 408.01
BD	2909 207	950 007.21	; DB	2913 668	946 872.89
CD	2913 100	943 273.69	; DC	2915 615	943 506.14

- -

Line	AC	AD	BC	BD	CD
$\xi_j - \xi_i$	− 4 322.18	+ 3 354.76	+ 5 201.83	+12 878.77	+ 7 676.94
$\eta_j - \eta_i$	−11 402.36	−10 705.03	−10 100.27	− 9 402.94	+ 697.33
$(T-t)_1$	−.002 9077	+.002 2574	+.003 4964	+.008 6584	+.005 1246
$(T-t)_2$	− 62	− 58	− 55	− 51	− 4
$(T-t)_3$	+ 214	− 166	− 257	− 636	− 371
$(T-t)_4$	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>
$(T-t)$	−.002 8925	+.002 2350	+.003 4652	+.008 5897	+.005 0879

Line	CA	DA	CB	DB	DC
$\xi_j-\xi_i$	+ 4 322.18	- 3 354.76	- 5 201.83	-12 878.77	- 7 676.94
$\eta_j-\eta_i$	+11 402.36	+10 705.03	+10 100.27	+ 9 402.94	- 697.33
$(T-t)_1$	+.002 8961	-.002 2490	-.003 4840	-.008 6298	-.005 1258
$(T-t)_2$	+ 61	+ 58	+ 55	+ 51	- 4
$(T-t)_3$	- 211	+ 164	+ 254	+ 630	+ 372
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	+.002 8811	-.002 2268	-.003 4531	-.008 5617	-.005 0890

$$-t'_{ij} + t'_{ik} = -ij + ik + (T-t)_{ij} - (T-t)_{ik}$$

$-t'_{AC} + t'_{AD} = a' = 38.159\ 740$ $\quad -t'_{CA} + t'_{CB} = e' = 48.009\ 095$

$-t'_{DA} + t'_{DB} = b' = 36.466\ 454$ $\quad -t'_{BC} + t'_{BD} = f' = 26.617\ 044$

$-t'_{DB} + t'_{DC} = c' = 41.323\ 791$ $\quad -t'_{BD} + t'_{BA} = g' = 43.918\ 634$

$-t'_{CD} + t'_{CA} = d' = 64.050\ 499$ $\quad -t'_{AB} + t'_{AC} = h' = 61.455\ 268$

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		v_1	v_2	v_3	
a'	38.159 740	-67	-13		$v_1 = -(a'+b'+\ldots h')/8$
					$v_2 = (a'+b'-(e'+f'))/4$
b'	36.466 454	-65	-14		subtract from a' and b',
c'	41.323 791	-65		-97	add to e' and f'.
d'	64.050 499	-66		-97	$v_3 = (c'+d'-(g'+h'))/4$
e'	48.009 095	-66	+14		subtract from c' and d',
f'	26.617 044	-65	+13		add to g' and h'.
g'	43.918 634	-65		+97	$v_4 = (\text{sum 1} - \text{sum 2})/\Sigma\delta$,
h'	61.455 268	-66		+97	subtract from a", c", e",
					g", add to other angles.

				v_4				
a"	38.159 660	a	38.159 695	35	b"	36.466 375	b	36.466 340
c"	41.323 629	c	41.323 664	35	d"	64.050 336	d	64.050 301
e"	48.009 043	e	48.009 078	35	f"	26.616 992	f	26.616 957
g"	43.918 666	g	43.918 701	35	h"	61.455 299	h	61.455 264

Log sin a"	9.790 8866	δ 9 6622	Log sin b"	9.774 0429	δ 10 2561
Log sin c"	9.819 7488	δ 8 6140	Log sin d"	9.953 8461	δ 3 6887
Log sin e"	9.871 1352	δ 6 8227	Log sin f"	9.651 3015	δ 15 1255
Log sin g"	9.841 1320	δ 7 8715	Log sin h"	9.943 7143	δ 4 1231
sum 1	9.322 9026		sum 2	9.322 9049	

$$
\begin{array}{lllll}
 & \bar{t}_{AB} = 187.785\ 020; & & & \bar{T}_{AB} = 187.778\ 637 \\
\bar{t}_{AB} + h & = \bar{t}_{AC} = 249.240\ 284; & \bar{t}_{AC} + (T-t)_{AC} & = & \bar{T}_{AC} = \\
\bar{t}_{AC} + a & = \bar{t}_{AD} = 287.399\ 979; & \bar{t}_{AD} + (T-t)_{AD} & = & \bar{T}_{AD} = \\
\bar{t}_{AD} + 180 & = \bar{t}_{DA} = 107.399\ 979; & \bar{t}_{DA} + (T-t)_{DA} & = & \bar{T}_{DA} = \\
\bar{t}_{DA} + b & = \bar{t}_{DB} = 143.866\ 319; & \bar{t}_{DB} + (T-t)_{DB} & = & \bar{T}_{DB} = \\
\bar{t}_{DB} + c & = \bar{t}_{DC} = 185.189\ 983; & \bar{t}_{DC} + (T-t)_{DC} & = & \bar{T}_{DC} = 185.184\ 894 \\
\bar{t}_{DC} + 180 & = \bar{t}_{CD} = 5.189\ 983; & \bar{t}_{CD} + (T-t)_{CD} & = & \bar{T}_{CD} = 5.195\ 071 \\
\bar{t}_{CD} + d & = \bar{t}_{CA} = 69.240\ 284; & \bar{t}_{CA} + (T-t)_{CA} & = & \bar{T}_{CA} = \\
\bar{t}_{CA} + e & = \bar{t}_{CB} = 117.249\ 362; & \bar{t}_{CB} + (T-t)_{CB} & = & \bar{T}_{CB} = \\
\bar{t}_{CB} + 180 & = \bar{t}_{BC} = 297.249\ 362; & \bar{t}_{BC} + (T-t)_{BC} & = & \bar{T}_{BC} = \\
\bar{t}_{BC} + f & = \bar{t}_{BD} = 323.866\ 319; & \bar{t}_{BD} + (T-t)_{BD} & = & \bar{T}_{BD} = \\
\bar{t}_{BD} + g & = \bar{t}_{BA} = 7.785\ 020; & & & \bar{T}_{BA} = 7.791\ 400
\end{array}
$$

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$$
\begin{array}{llllll}
\tan \bar{t}_{AD} & -3.1910\ 07979 & \tan \bar{t}_{DC} & +0.0908\ 30854 & \tan \bar{t}_{CB} & -1.9416\ 73094 \\
\tan \bar{t}_{BD} & \underline{-0.7301\ 13360} & \tan \bar{t}_{AC} & \underline{+2.6381\ 04519} & \tan \bar{t}_{DB} & \underline{-0.7301\ 13360} \\
\text{diff.} & -2.4608\ 94619 & \text{diff.} & -2.5472\ 73665 & \text{diff.} & -1.2115\ 59734
\end{array}
$$

$$\frac{(\eta_B-\eta_A) - \tan \bar{t}_{BD}(\xi_B-\xi_A)}{\tan \bar{t}_{AD} - \tan \bar{t}_{BD}} = \xi_D-\xi_A = +\ 3\ 354.75; \quad \bar{\xi}_D = 2950\ 586.28$$

$$\tan \bar{t}_{AD}(\xi_D-\xi_A) = \eta_D-\eta_A = -10\ 705.03; \quad \bar{\eta}_D = 943\ 738.58$$

$$\frac{\tan \bar{t}_{AC}(\xi_D-\xi_A) - (\eta_D-\eta_A)}{\tan \bar{t}_{AC} - \tan \bar{t}_{DC}} = \xi_C-\xi_D = -\ 7\ 676.92; \quad \bar{\xi}_C = 2942\ 909.36$$

$$\tan \bar{t}_{DC}(\xi_C-\xi_D) = \eta_C-\eta_D = -\ 697.30; \quad \bar{\eta}_C = 943\ 041.28$$

$$\frac{\tan \bar{t}_{DB}(\xi_C-\xi_D) - (\eta_C-\eta_D)}{\tan \bar{t}_{CB} - \tan \bar{t}_{DB}} = \xi_B-\xi_C = -\ 5\ 201.83; \quad \xi'_B = 2937\ 707.53$$

$$\tan \bar{t}_{CB}(\xi_B-\xi_C) = \eta_B-\eta_C = +10\ 100.25; \quad \eta'_B = 953\ 141.53$$

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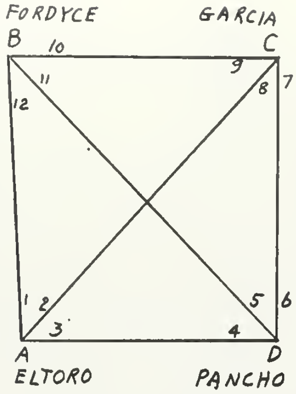
Stations: A. Palo; B. Pedro; C. Fordyce; D. Eltoro.

Datum: North American 1913; Ellipsoid: Clarke 1866; λ_o 108°W

ξ_B	2942 909.36
ξ_A	2950 586.28
$\xi_B - \xi_A$	- 7 676.92
$\overline{T}_{AB}$	185.184 894
-1 + 2	71.863 036
T'_{AC}	257.047 930
-2 + 3	44.655 253
T'_{AD}	301.703 183



η_B	943 041.28
η_A	943 738.58
$\eta_B - \eta_A$	- 697.30
$\overline{T}_{BA}$	5.195 071
+11 - 12	-43.085 647
T'_{BD}	322.109 424
+10 - 11	-33.714 261
T'_{BC}	288.395 163

$\tan T'_{AC}$	+4.3480 67709	$(T-t)_{AB}$	-.005 0890	$\tan T'_{AD}$	-1.6189 36744
$\tan T'_{BC}$	-3.0069 58258	$(T-t)_{BA}$	+.005 0879	$\tan T'_{BD}$	-0.7782 14607
diff	+7.3550 25967			diff	-0.8407 22137

$$\xi'_i - \xi_A = \frac{(\eta_B - \eta_A) - \tan T'_{Bi}(\xi_B - \xi_A)}{\tan T'_{Ai} - \tan T'_{Bi}}; \quad \eta'_i = \eta_A + \tan T'_{Ai}\ (\xi'_i - \xi_A).$$

$\xi'_C - \xi_A =$ -3 233.37; $\xi'_C =$ 2947 352.91; $\eta'_C =$ 929 679.65.

$\xi'_D - \xi_A =$ +7 935.56; $\xi'_D =$ 2958 521.84; $\eta'_D =$ 930 891.39.

$$\xi'_r = \xi'(1 - H_2\eta^2)$$

$\xi'_{rA} =$ 2918 130 $\quad \xi'_{rC} =$ 2915 891

$\xi'_{rB} =$ 2910 585 $\quad \xi'_{rD} =$ 2926 859

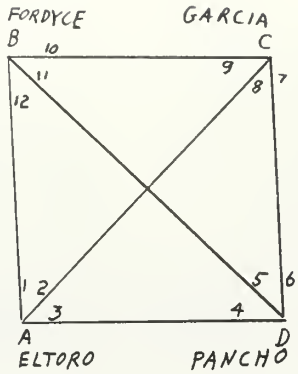
$$\xi_{3ij} = \frac{1}{3}\,(2\xi'_{ri} + \xi'_{rj}); \qquad \eta_{3ij} = \frac{1}{3}\,(2\eta'_i + \eta'_j)$$

	ξ_3	η_3		ξ_3	η_3
AC	2917 384	939 052.26	; BC	2912 354	938 587.39
AD	2921 040	939 456.17	; BD	2916 010	938 991.30

$$(T-t) = I_1(\xi_j - \xi_i)\eta_3 + I_2(\eta_j - \eta_i)\eta_3^2 - I_3(\xi_j - \xi_i)\eta_3^3 - I_4(\eta_j - \eta_i)\eta_3^4$$

Line	AC	AD	BC	BD
$\xi_j - \xi_i$	- 3 233.37	+ 7 935.56	+ 4 443.56	+15 612.49
$\eta_j - \eta_i$	-14 058.91	-12 847.17	-13 361.61	-12 149.87
$(T-t)_1$	-.002 1490	+.005 2758	+.002 9515	+.010 3744
$(T-t)_2$	- 75	- 68	- 71	- 64
$(T-t)_3$	+ 154	- 379	- 212	- 745
$(T-t)_4$	negligible	negligible	negligible	negligible
$(T-t)$	-.002 1411	+.005 2311	+.002 9232	+.010 2935

ξ_B	2942 909.36
ξ_A	2950 586.28
$\xi_B - \xi_A$	- 7 676.92
$\bar{t}_{AB}$	185.189 983
-1' + 2'	71.860 088
t''_{AC}	257.050 071
-2' + 3'	44.647 881
t''_{AD}	301.697 952



η_B	943 041.28
η_A	943 738.58
$\eta_B - \eta_A$	- 697.30
$\bar{t}_{BA}$	5.189 983
+11' - 12'	-43.090 853
t''_{BD}	322.099 130
+10' - 11'	-33.706 841
t''_{BC}	288.392 239

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$\tan t''_{AC}$	+4.3488 11308	$(T-t)_{AB}$	-.005 0890	$\tan t''_{AD}$	-1.6192 67443
$\tan t''_{BC}$	-3.0074 70979	$(T-t)_{BA}$	+.005 0879	$\tan t''_{BD}$	-0.7785 03147
diff.	+7.3562 82287			diff.	-0.8407 64296

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$-1' + 2' = -1 + 2 + (T-t)_{AB} - (T-t)_{AC}$ etc.

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$\xi''_C - \xi_A =$ -3 233.35; $\xi''_C =$ 2947 352.93; $\eta''_C =$ 929 677.35.

$\xi''_D - \xi_A =$ +7 937.79; $\xi''_D =$ 2958 524.07; $\eta''_C =$ 930 885.18.

$\xi''_r = \xi''(1 - H_2\eta^2)$ $\xi''_{rA} =$ 2918 130 $\xi''_{rC} =$ 2915 891

$\xi''_{rB} =$ 2910 585 $\xi''_{rD} =$ 2926 862

	ξ_3	η_3			ξ_3	η_3
AC	2917 384	939 051.50	;	CA	2916 137	934 364.42
AD	2921 041	939 454.11	;	DA	2923 951	935 169.65
BC	2912 354	938 586.64	;	CB	2914 122	934 131.99
BD	2916 011	938 989.25	;	DB	2921 436	934 937.21
CD	2919 548	930 079.96	;	DC	2923 205	930 482.57

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Line	AC	AD	BC	BD	CD
$\xi_j - \xi_i$	- 3 233.35	+ 7 937.79	+ 4 443.57	+15 614.71	+11 171.14
$\eta_j - \eta_i$	-14 061.23	-12 853.40	-13 363.93	-12 156.10	+ 1 207.83
$(T-t)_1$	-.002 1487	+.005 2772	+.002 9515	+.010 3759	+.007 3527
$(T-t)_2$	- 75	- 68	- 71	- 64	+ 6
$(T-t)_3$	+ 154	- 379	- 212	- 745	- 518
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	-.002 1408	+.005 2325	+.002 9232	+.010 2950	+.007 3015

Line	CA	DA	CB	DB	DC
$\xi_j-\xi_i$	+ 3 233.35	- 7 937.79	- 4 443.57	-15 614.71	-11 171.14
$\eta_j-\eta_i$	+14 061.23	+12 853.40	+13 363.93	+12 156.10	- 1 207.83
$(T-t)_1$	+.002 1382	-.005 2531	-.002 9375	-.010 3311	-.007 3558
$(T-t)_2$	+ 74	+ 68	+ 71	+ 64	- 6
$(T-t)_3$	- 152	+ 374	+ 209	+ 735	+ 519
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	+.002 1304	-.005 2089	-.002 9095	-.010 2512	-.007 3045

$$-t'_{ij} + t'_{ik} = -ij + ik + (T-t)_{ij} - (T-t)_{ik}$$

$-t'_{AC} + t'_{AD} = a' = 44.647\ 880$ $\quad$ $-t'_{CA} + t'_{CB} = e' = 31.342\ 099$

$-t'_{DA} + t'_{DB} = b' = 20.401\ 373$ $\quad$ $-t'_{BC} + t'_{BD} = f' = 33.706\ 889$

$-t'_{DB} + t'_{DC} = c' = 44.072\ 005$ $\quad$ $-t'_{BD} + t'_{BA} = g' = 43.090\ 854$

$-t'_{CD} + t'_{CA} = d' = 70.878\ 793$ $\quad$ $-t'_{AB} + t'_{AC} = h' = 71.860\ 088$

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		v_1	v_2	v_3
a'	44.647 880	+2	-66	
b'	20.401 373	+2	-66	
c'	44.072 005	+3		+37
d'	70.878 793	+2		+35
e'	31.342 099	+2	+66	
f'	33.706 889	+3	+66	
g'	43.090 854	+2		-36
h'	71.860 088	+3		-36

$v_1 = -(a'+b'+\ldots h')/8$

$v_2 = (a'+b'-(e'+f'))/4$ subtract from a' and b', add to e' and f'.

$v_3 = (c'+d'-(g'+h'))/4$ subtract from c' and d', add to g' and h'.

$v_4 = (\text{sum } 1 - \text{sum } 2)/\Sigma\delta$, subtract from a", c", e" and g", add to others.

		v_4		
a" 44.647 816	a 44.647 820	4	b" 20.401 309	b 20.401 305
c" 44.072 045	c 44.072 049	4	d" 70.878 830	d 70.878 826
e" 31.342 167	e 31.342 171	4	f" 33.706 958	f 33.706 954
g" 43.090 820	g 43.090 824	4	h" 71.860 055	h 71.860 051

Log sin a"	9.846 7991	δ 7 6736	Log sin b"	9.542 3192	δ 20 3802
Log sin c"	9.842 3360	δ 7 8295	Log sin d"	9.975 3527	δ 2 6279
Log sin e"	9.716 1267	δ 12 4461	Log sin f"	9.744 2504	δ 11 3625
Log sin g"	9.834 5204	δ 8 1025	Log sin h"	9.977 8602	δ 2 4833
sum 1	9.239 7822		sum 2	9.239 7825	

	$\bar{t}_{AB}$	= 185.189 983		$\bar{T}_{AB}$	= 185.184 894
$\bar{t}_{AB}$ + h	= $\bar{t}_{AC}$	= 257.050 034;	$\bar{t}_{AC}$ +$(T-t)_{AC}$	= $\bar{T}_{AC}$	=
$\bar{t}_{AC}$ + a	= $\bar{t}_{AD}$	= 301.697 854;	$\bar{t}_{AD}$ +$(T-t)_{AD}$	= $\bar{T}_{AD}$	=
$\bar{t}_{AD}$ + 180	= $\bar{t}_{DA}$	= 121.697 854;	$\bar{t}_{DA}$ +$(T-t)_{DA}$	= $\bar{T}_{DA}$	=
$\bar{t}_{DA}$ + b	= $\bar{t}_{DB}$	= 142.099 159;	$\bar{t}_{DB}$ +$(T-t)_{DB}$	= $\bar{T}_{DB}$	=
$\bar{t}_{DB}$ + c	= $\bar{t}_{DC}$	= 186.171 208;	$\bar{t}_{DC}$ +$(T-t)_{DC}$	= $\bar{T}_{DC}$	= 186.163 902
$\bar{t}_{DC}$ + 180	= $\bar{t}_{CD}$	= 6.171 208;	$\bar{t}_{CD}$ +$(T-t)_{CD}$	= $\bar{T}_{CD}$	= 6.178 510
$\bar{t}_{CD}$ + d	= $\bar{t}_{CA}$	= 77.050 034;	$\bar{t}_{CA}$ +$(T-t)_{CA}$	= $\bar{T}_{CA}$	=
$\bar{t}_{CA}$ + e	= $\bar{t}_{CB}$	= 108.392 205;	$\bar{t}_{CB}$ +$(T-t)_{CB}$	= $\bar{T}_{CB}$	=
$\bar{t}_{CB}$ + 180	= $\bar{t}_{BC}$	= 288.392 205;	$\bar{t}_{BC}$ +$(T-t)_{BC}$	= $\bar{T}_{BC}$	=
$\bar{t}_{BC}$ + f	= $\bar{t}_{BD}$	= 322.099 159;	$\bar{t}_{BD}$ +$(T-t)_{BD}$	= $\bar{T}_{BD}$	=
$\bar{t}_{BD}$ + g	= $\bar{t}_{BA}$	= 5.189 983		$\bar{T}_{BA}$	= 5.195 071

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tan $\bar{t}_{AD}$	-1.6192 73575	tan $\bar{t}_{DC}$	+0.1081 26346	tan $\bar{t}_{CB}$	-3.0074 76940
tan $\bar{t}_{BD}$	-0.7785 02274	tan $\bar{t}_{AC}$	+4.3487 98449	tan $\bar{t}_{DB}$	-0.7785 02274
diff.	-0.8407 71301	diff.	-4.2406 72103	diff.	-2.2289 74666

$$\frac{(\eta_B-\eta_A) - \tan \bar{t}_{BD}(\xi_B-\xi_A)}{\tan \bar{t}_{AD} - \tan \bar{t}_{BD}} = \xi_D-\xi_A = +\ 7\ 937.72; \quad \bar{\xi}_D = 2958\ 524.00$$

$$\tan \bar{t}_{AD}(\xi_D-\xi_A) = \eta_D-\eta_A = -12\ 853.34; \quad \bar{\eta}_D = 930\ 885.24$$

$$\frac{\tan \bar{t}_{AC}(\xi_D-\xi_A) - (\eta_D-\eta_A)}{\tan \bar{t}_{AC} - \tan \bar{t}_{DC}} = \xi_C-\xi_D = -11\ 171.08; \quad \bar{\xi}_C = 2947\ 352.92$$

$$\tan \bar{t}_{DC}(\xi_C-\xi_D) = \eta_C-\eta_D = -\ 1\ 207.89; \quad \bar{\eta}_C = 929\ 677.35$$

$$\frac{\tan \bar{t}_{DB}(\xi_C-\xi_D) - (\eta_C-\eta_D)}{\tan \bar{t}_{CB} - \tan \bar{t}_{DB}} = \xi_B-\xi_C = -\ 4\ 443.57; \quad \xi'_B = 2942\ 909.35$$

$$\tan \bar{t}_{CB}(\xi_B-\xi_C) = \eta_B-\eta_C = +13\ 363.93; \quad \eta'_B = 943\ 041.28$$

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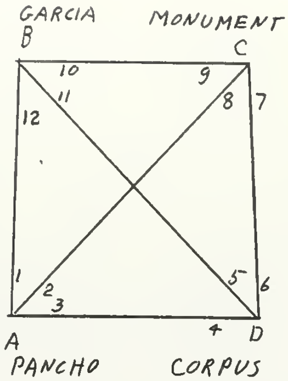
Stations: A. Eltoro; B. Fordyce; C. Garcia; D. Pancho.

Datum: North American 1913; Ellipsoid: Clarke 1866; λ_o 108°W



ξ_B	2947 353.92	η_B	929 677.35
ξ_A	2958 524.00	η_A	930 885.24
$\xi_B - \xi_A$	-11 171.08	$\eta_B - \eta_A$	-1 207.89
$\overline{T}_{AB}$	186.163 905	$\overline{T}_{BA}$	6.178 508
-1 + 2	28.467 561	+11 - 12	-38.637 716
T'_{AC}	214.631 466	T'_{BD}	327.540 792
-2 + 3	49.310 942	+10 - 11	-51 259 306
T'_{AD}	263.942 408	T'_{BC}	276.281 486

$\tan T'_{AC}$	+0.6906 64641	$(T-t)_{AB}$	-.007 3045	$\tan T'_{AD}$	+9.4232 39586
$\tan T'_{BC}$	-9.0847 99909	$(T-t)_{BA}$	+.007 3015	$\tan T'_{BD}$	-0.6360 69807
diff	+9.7754 64550			diff	+10.0593 0939

$$\xi'_i - \xi_A = \frac{(\eta_B - \eta_A) - \tan T'_{Bi}(\xi_B - \xi_A)}{\tan T'_{Ai} - \tan T'_{Bi}}; \quad \eta'_i = \eta_A + \tan T'_{Ai}\,(\xi'_i - \xi_A).$$

$\xi'_C - \xi_A =$ -10.505.37; $\xi'_C =$ 2948 018.63; $\eta'_C =$ 923 629.53.

$\xi'_D - \xi_A =$ - 826.45; $\xi'_D =$ 2957 697.55; $\eta'_D =$ 923 097.38.

$\xi'_r = \xi'(1 - H_2\eta^2)$

ξ'_{rA}	2926 859	ξ'_{rC} =	2916 958
ξ'_{rB}	2915 891	ξ'_{rD} =	2926 535

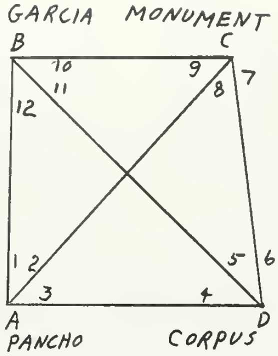
$$\xi_{3ij} = \frac{1}{3}(2\xi'_{ri} + \xi'_{rj}); \qquad \eta_{3ij} = \frac{1}{3}(2\eta'_i + \eta'_j)$$

	ξ_3	η_3		ξ_3	η_3
AC	2923 559	928 466.65 ;	BC	2916 247	927 661.40
AD	2926 751	928 289.27 ;	BD	2919 439	927 484.01

$$(T-t) = I_1(\xi_j - \xi_i)\eta_3 + I_2(\eta_j - \eta_i)\eta_3^2 - I_3(\xi_j - \xi_i)\eta_3^3 - I_4(\eta_j - \eta_i)\eta_3^4$$

Line	AC	AD	BC	BD
$\xi_j - \xi_i$	-10 505.37	- 826.45	+ 665.71	+10 344.63
$\eta_j - \eta_i$	- 7 255.69	- 7 787.84	- 6 047.80	- 6 579.95
$(T-t)_1$	-.006 9025	-.000 5429	+.000 4370	+.006 7897
$(T-t)_2$	- 38	- 40	- 31	- 34
$(T-t)_3$	+ 485	+ 38	- 31	- 476
$(T-t)_4$	negligible	negligible	negligible	negligible
$(T-t)$	-.006 8578	-.000 5431	+.000 4308	+.006 7387

ξ_B	2947 352.92	η_B	929 677.35
ξ_A	<u>2958 524.00</u>	η_A	<u>930 885.24</u>
$\xi_B - \xi_A$	-11 171.08	$\eta_B - \eta_A$	- 1 207.89
$\bar{t}_{AB}$	186.171 208	$\bar{t}_{BA}$	6.171 208
-1' + 2'	<u>28.467 114</u>	+11' - 12'	<u>-38.637 155</u>
t''_{AC}	214.638 322	t''_{BD}	327.534 053
-2' + 3'	<u>49.304 627</u>	+10' - 11'	<u>-51.252 998</u>
t''_{AD}	263.942 949	t''_{BC}	276.281 055



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tan t''_{AC}	+0.6908 41395	$(T-t)_{AB}$	-.007 3045	tan t''_{AD}	+9.4240 87580
tan t''_{BC}	<u>-9.0854 28322</u>	$(T-t)_{BA}$	+.007 3015	tan t''_{BD}	<u>-0.6362 35023</u>
diff.	+9.7762 69717			diff.	+10.0603 22603

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$-1' + 2' = -1 + 2 + (T-t)_{AB} - (T-t)_{AC}$ etc.

- -

$\xi''_C - \xi_A =$ -10 505.23; $\xi''_C =$ 2948 018.77; $\eta''_C =$ 923 627.79.

$\xi''_D - \xi_A =$ - 826.55; $\xi''_D =$ 2957 697.45; $\eta''_D =$ 923 095.76.

$\xi''_r = \xi''(1 - H_2\eta^2)$

$\xi''_{rA} =$	2926 862	$\xi''_{rC} =$	2916 958
$\xi''_{rB} =$	2915 891	$\xi''_{rD} =$	2926 571

	ξ_3	η_3		ξ_3	η_3
AC	2923 559	928 466.09	; CA	2920 256	926 046.94
AD	2926 763	928 288.75	; DA	2926 667	925 692.25
BC	2916 247	927 660.83	; CB	2916 602	925 644.31
BD	2919 451	927 483.49	; DB	2923 011	925 289.62
CD	2920 162	923 450.45	; DC	2923 367	923 273.10

- -

Line	AC	AD	BC	BD	CD
$\xi_j - \xi_i$	-10 505.23	- 826.55	+ 665.85	+10 344.53	+ 9 678.68
$\eta_j - \eta_i$	- 7 257.45	- 7 789.48	- 6 049.56	- 6 581.59	- 532.03
$(T-t)_1$	-.006 9024	-.000 5430	+.000 4371	+.006 7896	+.006 3250
$(T-t)_2$	- 38	- 40	- 31	- 34	- 3
$(T-t)_3$	+ 485	+ 38	- 31	- 476	- 439
$(T-t)_4$	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>
$(T-t)$	-.006 8577	-.000 5432	+.000 4309	+.006 7386	+.006 2808

Line	CA	DA	CB	DB	DC
$\xi_j-\xi_i$	+10 505.23	+ 826.55	- 665.85	-10 344.53	- 9 678.68
$\eta_j-\eta_i$	+ 7 257.45	+ 7 789.48	+ 6 049.56	+ 6 581.59	+ 532.03
$(T-t)_1$	+.006 8844	+.000 5415	-.000 4362	-.006 7735	-.006 3237
$(T-t)_2$	+ 38	+ 40	+ 31	+ 34	+ 3
$(T-t)_3$	- 481	- 38	+ 31	+ 472	+ 439
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	+.006 8401	+.000 5417	-.000 4300	-.006 7229	-.006 2795

$-t'_{ij} + t'_{ik} = -ij + ik + (T-t)_{ij} - (T-t)_{ik}$

$-t'_{AC} + t'_{AD} = a' = 49.304\ 628$ $\quad -t'_{CA} + t'_{CB} = e' = 61.643\ 026$

$-t'_{DA} + t'_{DB} = b' = 63.591\ 629$ $\quad -t'_{BC} + t'_{BD} = f' = 51.252\ 998$

$-t'_{DB} + t'_{DC} = c' = 29.319\ 612$ $\quad -t'_{BD} + t'_{BA} = g' = 38.637\ 153$

$-t'_{CD} + t'_{CA} = d' = 37.784\ 602$ $\quad -t'_{AB} + t'_{AC} = h' = 28.467\ 114$

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		v_1	v_2	v_3
a'	49.304 628	-95	-58	
b'	63.591 629	-96	-58	
c'	29.319 612	-95		+13
d'	37.784 602	-95		+13
e'	61.643 026	-95	+58	
f'	51.252 998	-95	+58	
g'	38.637 153	-95		-13
h'	28.467 114	-96		-13

$v_1 = -(a'+b'+\ldots h')/8$

$v_2 = (a'+b'-(e'+f'))/4$

subtract from a' and b', add to e' and f'.

$v_3 = (c'+d'-(g'+h'))/4$

subtract from c' and d', add to g' and h'.

$v_4 = (\text{sum 1} - \text{sum 2})/\Sigma\delta$,

subtract from a", c", e", g", add to other angles.

		v_4		
a" 49.304 475	a 49.304 546	71	b" 63.591 475	b 63.591 404
c" 29.319 530	c 29.319 601	71	d" 37.784 520	d 37.784 449
e" 61.642 989	e 61.643 060	71	f" 51.252 961	f 51.252 890
g" 38.637 045	g 38.637 116	71	h" 28.467 005	h 28.466 934

Log sin a"	9.879 7754	δ 6 5188	Log sin b"	9.952 1362	δ 3 7641
Log sin c"	9.689 9121	δ 13 4963	Log sin d"	9.787 2433	δ 9 7774
Log sin e"	9.944 4852	δ 4 0910	Log sin f"	9.892 0483	δ 6 0828
Log sin g"	9.795 4523	δ 9 4825	Log sin h"	9.678 2020	δ 13 9797
sum 1	9.309 6250		sum 2	9.309 6298	

$$
\begin{array}{lllll}
 & \bar{t}_{AB} & = 186.171\ 208; & & \bar{T}_{AB} = 186.163\ 905 \\
\bar{t}_{AB} + h & = \bar{t}_{AC} & = 214.638\ 142; & \bar{t}_{AC} + (T-t)_{AC} & = \bar{T}_{AC} = \\
\bar{t}_{AC} + a & = \bar{t}_{AD} & = 263.942\ 688; & \bar{t}_{AD} + (T-t)_{AD} & = \bar{T}_{AD} = \\
\bar{t}_{AD} + 180 & = \bar{t}_{DA} & = 83.942\ 688; & \bar{t}_{DA} + (T-t)_{DA} & = \bar{T}_{DA} = \\
\bar{t}_{DA} + b & = \bar{t}_{DB} & = 147.534\ 092; & \bar{t}_{DB} + (T-t)_{DB} & = \bar{T}_{DB} = \\
\bar{t}_{DB} + c & = \bar{t}_{DC} & = 176.853\ 693; & \bar{t}_{DC} + (T-t)_{DC} & = \bar{T}_{DC} = 176.847\ 414 \\
\bar{t}_{DC} + 180 & = \bar{t}_{CD} & = 356.853\ 693; & \bar{t}_{CD} + (T-t)_{CD} & = \bar{T}_{CD} = 356.859\ 974 \\
\bar{t}_{CD} + d & = \bar{t}_{CA} & = 34.638\ 142; & \bar{t}_{CA} + (T-t)_{CA} & = \bar{T}_{CA} = \\
\bar{t}_{CA} + e & = \bar{t}_{CB} & = 96.281\ 202; & \bar{t}_{CB} + (T-t)_{CB} & = \bar{T}_{CB} = \\
\bar{t}_{CB} + 180 & = \bar{t}_{BC} & = 276.281\ 202; & \bar{t}_{BC} + (T-t)_{BC} & = \bar{T}_{BC} = \\
\bar{t}_{BC} + f & = \bar{t}_{BD} & = 327.534\ 092; & \bar{t}_{BD} + (T-t)_{BD} & = \bar{T}_{BD} = \\
\bar{t}_{BD} + g & = \bar{t}_{BA} & = 6.171\ 208; & & \bar{T}_{BA} = 6.178\ 508
\end{array}
$$

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tan $\bar{t}_{AD}$	+9.4236 78443	tan $\bar{t}_{DC}$	-0.0549 68680	tan $\bar{t}_{CB}$	-9.0852 13983
tan $\bar{t}_{BD}$	<u>-0.6362 34067</u>	tan $\bar{t}_{AC}$	<u>+0.6908 36754</u>	tan $\bar{t}_{DB}$	<u>-0.6362 34067</u>
diff.	+10.0599 1251	diff.	-0.7458 05434	diff.	-8.4489 79916

$$\frac{(\eta_B-\eta_A) - \tan \bar{t}_{BD}(\xi_B-\xi_A)}{\tan \bar{t}_{AD} - \tan \bar{t}_{BD}} = \xi_D-\xi_A = -\ 826.58; \quad \bar{\xi}_D = 2957\ 697.42$$

$$\tan \bar{t}_{AD}(\xi_D-\xi_A) = \eta_D-\eta_A = -\ 7\ 789.42; \quad \bar{\eta}_D = 923\ 095.82$$

$$\frac{\tan \bar{t}_{AC}(\xi_D-\xi_A) - (\eta_D-\eta_A)}{\tan \bar{t}_{AC} - \tan \bar{t}_{DC}} = \xi_C-\xi_D = -\ 9\ 678.65; \quad \bar{\xi}_C = 2948\ 018.77$$

$$\tan \bar{t}_{DC}(\xi_C-\xi_D) = \eta_C-\eta_D = +\ 532.02; \quad \bar{\eta}_C = 923\ 627.84$$

$$\frac{\tan \bar{t}_{DB}(\xi_C-\xi_D) - (\eta_C-\eta_D)}{\tan \bar{t}_{CB} - \tan \bar{t}_{DB}} = \xi_B-\xi_C = -\ 665.86; \quad \xi_B' = 2947\ 352.91$$

$$\tan \bar{t}_{CB}(\xi_B-\xi_C) = \eta_B-\eta_C = +\ 6\ 049.51; \quad \eta_B' = 929\ 677.35$$

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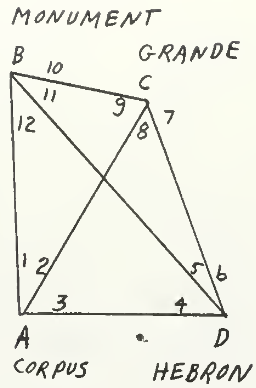
Stations: A. Pancho; B. Garcia; C. Monument; D. Corpus.

Datum: North American 1913; Ellipsoid: Clarke 1866; λ_o 108°W

ξ_B	2948 018.77	η_B	923 627.84
ξ_A	<u>2957 697.92</u>	η_A	<u>923 095.82</u>
$\xi_B - \xi_A$	- 9 678.65	$\eta_B - \eta_A$	+ 532.02
$\overline{T}_{AB}$	176.847 414	$\overline{T}_{BA}$	356.859 974
-1 + 2	<u>46.299 759</u>	+11 - 12	<u>-48.746 908</u>
T'_{AC}	223.147 173	T'_{BD}	308.113 066
-2 + 3	<u>47.512 066</u>	+10 - 11	<u>- 6.792 311</u>
T'_{AD}	270.659 239	T'_{BC}	301.320 755



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$\tan T'_{AC}$	+0.9373 28942	$(T-t)_{AB}$	-.006 2795	$\tan T'_{AD}$	-86.908 16376
$\tan T'_{BC}$	<u>-1.6433 69723</u>	$(T-t)_{BA}$	+.006 2808	$\tan T'_{BD}$	<u>- 1.274 74849</u>
diff	+2.5806 98665			diff	-85.633 41527

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$$\xi'_i - \xi_A = \frac{(\eta_B - \eta_A) - \tan T'_{Bi}(\xi_B - \xi_A)}{\tan T'_{Ai} - \tan T'_{Bi}}; \qquad \eta'_i = \eta_A + \tan T'_{Ai}\ (\xi'_i - \xi_A).$$

$\xi'_C - \xi_A = -5\ 957.14$; $\xi'_C = 2951\ 740.28$; $\eta'_C = 917\ 512.02$.

$\xi'_D - \xi_A = +\ 137.86$; $\xi'_D = 2957\ 835.28$; $\eta'_D = 911\ 114.25$.

$$\xi'_r = \xi'(1 - H_2\eta^2)$$

$\xi'_{rA} =$ 2926 571 $\qquad \xi'_{rC} =$ 2921 051

$\xi'_{rB} =$ 2916 958 $\qquad \xi'_{rD} =$ 2927 510

$$\xi_{3ij} = \frac{1}{3}(2\xi'_{ri} + \xi'_{rj}); \qquad \eta_{3ij} = \frac{1}{3}(2\eta'_i + \eta'_j)$$

	ξ_3	η_3		ξ_3	η_3
AC	2924 731	921 234.55	; BC	2918 322	921 589.23
AD	2926 884	919 101.96	; BD	2920 475	919 456.43

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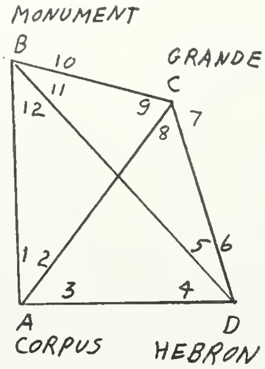
$$(T-t) = I_1(\xi_j - \xi_i)\eta_3 + I_2(\eta_j - \eta_i)\eta_3^2 - I_3(\xi_j - \xi_i)\eta_3^3 - I_4(\eta_j - \eta_i)\eta_3^4$$

Line	AC	AD	BC	BD
$\xi_j - \xi_i$	- 5 957.14	+ 137.86	+ 3 721.51	+ 9 816.51
$\eta_j - \eta_i$	- 5 583.80	-11 981.57	- 6 115.82	-12 513.59
$(T-t)_1$	-.003 8836	+.000 0897	+.002 4271	+.006 3873
$(T-t)_2$	- 29	- 61	- 31	- 64
$(T-t)_3$	+ 268	- 8	- 168	- 440
$(T-t)_4$	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>	<u>negligible</u>
$(T-t)$	-.003 8597	+.000 0828	+.002 4072	+.006 3369

ξ_B	2948 018.77
ξ_A	2957 697.42
$\xi_B - \xi_A$	− 9 678.65
$\bar{t}_{AB}$	176.853 693
$-1' + 2'$	46.297 339
t''_{AC}	223.151 032
$-2' + 3'$	47.508 124
t''_{AD}	270.659 156



η_B	923 627.84
η_A	923 095.82
$\eta_B - \eta_A$	+ 532.02
$\bar{t}_{BA}$	356.853 693
$+11'-12'$	−48.746 964
t''_{BD}	308.106 729
$+10' - 11'$	− 6.788 381
t''_{BC}	301.318 348

$\tan t''_{AC}$	+0.9374 55477	$(T-t)_{AB}$	−.006 2795	$\tan t''_{AD}$	−86.919 10858
$\tan t''_{BC}$	−1.6435 25199	$(T-t)_{BA}$	+.006 2808	$\tan t''_{BD}$	− 1.275 03886
diff.	+2.5809 80676			diff.	−85.644 06972

$-1' + 2' = -1 + 2 + (T-t)_{AB} - (T-t)_{AC}$ etc.

$\xi''_C - \xi_A = -5\ 957.07$; $\xi''_C = 2951\ 740.35$; $\eta''_C = 917\ 511.33$.

$\xi''_D - \xi_A = +\ 137.88$; $\xi''_D = 2957\ 835.30$; $\eta''_D = 911\ 111.38$.

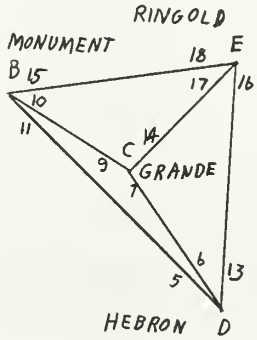
$\xi''_r = \xi''(1 - H_2\eta^2)$ $\quad \xi''_{rA} = 2926\ 571 \quad \xi''_{rC} = 2921\ 051$

$\xi''_{rB} = 2916\ 958 \quad \xi''_{rD} = 2927\ 510$

	ξ_3	η_3		ξ_3	η_3
AC	2924 731	921 234.32	; CA	2922 891	919 372.83
AD	2926 884	919 101.01	; DA	2927 197	915 106.19
BC	2918 322	921 589.00	; CB	2919 687	919 550.17
BD	2920 475	919 455.69	; DB	2923 993	915 283.53
CD	2923 204	915 378.01	; DC	2925 357	913 244.70

Line	AC	AD	BC	BD	CD
$\xi_j - \xi_i$	− 5 957.07	+ 137.88	+ 3 721.58	+ 9 816.53	+ 6 094.95
$\eta_j - \eta_i$	− 5 584.49	−11 984.44	− 6 116.51	−12 516.46	− 6 399.95
$(T-t)_1$	−.003 8835	+.000 0897	+.002 4271	+.006 3873	+.003 9482
$(T-t)_2$	− 29	− 61	− 31	− 64	− 32
$(T-t)_3$	+ 268	− 6	− 168	− 440	− 269
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	−.003 8596	+.000 0830	+.002 4072	+.006 3369	+.003 9181

Line	CA	DA	CB	DB	DC
$\xi_j-\xi_i$	+ 5 957.07	- 137.88	- 3 721.58	- 9 816.53	- 6 094.95
$\eta_j-\eta_i$	+ 5 584.49	+11 984.44	+ 6 116.51	+12 516.46	+ 6 399.95
$(T-t)_1$	+.003 8757	-.000 0893	-.002 4218	-.006 3583	-.003 9390
$(T-t)_2$	+ 28	+ 60	+ 31	+ 63	+ 32
$(T-t)_3$	- 267	+ 6	+ 167	+ 434	+ 268
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	+.003 8518	-.000 0827	-.002 4020	-.006 3086	-.003 9090



ξ_B	2948 018.77	η_B	923 627.84
ξ_D	2957 835.30	η_D	911 111.38
$\xi_B - \xi_D$	- 9 816.53	$\eta_B - \eta_D$	+12 516.46
$\bar{t}_{DB}$	128.106 729	$\bar{t}_{BD}$	308.106 729
$\bar{T}_{DB}$	128.100 392	$\bar{T}_{BD}$	308.113 066
-5 + 13	52.875 495	+15 - 11	-31.799 875
T'_{DE}	180.975 887	T'_{BE}	276.313 191

tan T'_{DE}	+0.0170 34089	$(T-t)_{DB}$	-.006 3086
tan T'_{BE}	-9.0388 07244	$(T-t)_{BD}$	+.006 3369
diff.	+9.0558 41333		

$\xi'_E-\xi'_D$ = - 8 415.92; ξ'_E = 2949 419.38; η'_E = 910 968.02

ξ'_{rD} = 2927 510

ξ'_{rB} = 2916 958

ξ'_{rE} = 2919 190

DE ξ_3 2924 737, η_3 911 063.26;

BE ξ_3 2917 702, η_3 919 407.57.

Line	DE	BE
$\xi_j-\xi_i$	- 8 415.92	+ 1 400.61
$\eta_j-\eta_i$	- 143.36	-12 659.82
$(T-t)_1$	-.005 4259	+.000 9113
$(T-t)_2$	- 1	- 65
$(T-t)_3$	+ 366	- 63
$(T-t)_4$	negligible	negligible
$(T-t)$	-.005 3894	+.000 8985

$\bar{t}_{DB}$	128.106 729	$\bar{t}_{BD}$	308.106 729
-5' + 13'	52.874 576	+15' - 11'	-31.794 437
t''_{DE}	180.981 305	t''_{BE}	276.312 292

tan t''_{DE}	+0.0171 28678	$(T-t)_{DB}$	-.006 3086
tan t''_{BE}	-9.0401 05040	$(T-t)_{BD}$	+.006 3369
diff.	+9.0572 33718		

$\xi'_E - \xi'_D = -\ 8\ 416.04;$ $\xi'_E = 2949\ 419.26;$ $\eta'_E = 910\ 967.22$

$\xi'_{rD} = 2927\ 510$ $\xi'_{rB} = 2916\ 958$ $\xi'_{rE} = 2919\ 190$

	ξ_3	η_3		ξ_3	η_3
DE	2924 737	911 063.32	; ED	2921 963	911 015.27
BE	2917 702	919 407.63	; EB	2917 924	915 187.42
CE	2920 431	915 329.95	; EC	2919 810	913 148.58

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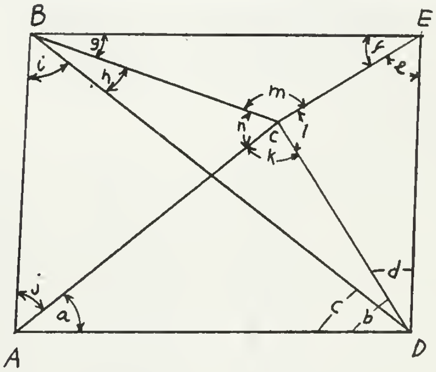
Line	DE	ED	BE	EB
$\xi_j - \xi_i$	- 8 416.04	+ 8 416.04	+ 1 400.49	- 1 400.49
$\eta_j - \eta_i$	- 144.16	+ 144.16	-12 660.62	+12 660.62
$(T-t)_1$	-.005 4260	+.005 4258	+.000 9112	-.000 9070
$(T-t)_2$	- 1	+ 1	- 65	+ 64
$(T-t)_3$	+ 367	- 367	- 63	+ 62
$(T-t)_4$	negligible	negligible	negligible	negligible
$(T-t)$	-.005 3894	+.005 3892	+.000 8984	-.000 8944

Line	CE	EC
$\xi_j - \xi_i$	- 2 321.09	+ 2 321.09
$\eta_j - \eta_i$	- 6 544.11	- 6 544.11
$(T-t)_1$	-.001 5035	+.001 4999
$(T-t)_2$	- 33	+ 33
$(T-t)_3$	+ 102	- 101
$(T-t)_4$	negligible	negligible
$(T-t)$	-.001 4966	+.001 4931



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$-t'_{ij} + t'_{ik} = -ij + ik + (T-t)_{ij} - (T-t)_{ik}$

$-t'_{AC} + t'_{AD} = a' = 47.508\ 123$	$-t'_{BC} + t'_{BA} = i' = 55.535\ 345$
$-t'_{DA} + t'_{DC} = b' = 42.942\ 848$	$-t'_{AB} + t'_{AC} = j' = 46.297\ 339$
$-t'_{DC} + t'_{DE} = d' = 47.379\ 772$	$-t'_{CD} + t'_{CA} = k' = 89.549\ 402$
$-t'_{ED} + t'_{EC} = e' = 69.489\ 238$	$-t'_{CE} + t'_{CD} = l' = 63.131\ 060$
$-t'_{EC} + t'_{EB} = f' = 25.841\ 618$	$-t'_{CB} + t'_{CE} = m' = 129.152\ 137$
$-t'_{BE} + t'_{BC} = g' = 25.006\ 055$	$-t'_{CA} + t'_{CB} = n' = 78.167\ 401$
$-t'_{DA} + t'_{DB} = c' = 37.448\ 045$	$-t'_{BC} + t'_{BD} = h' = 6.788\ 381$

		v_1			v_1
a'	47.508 123	-187	d'	47.379 772	- 35
b'	42.942 848	-186	e'	69.489 238	- 35
k'	89.549 402	0	l'	63.131 060	0
f'	25.841 618	+ 95	i'	55.535 345	- 43
g'	25.006 055	+ 95	j'	46.297 339	- 42
m'	129.152 137	0	n'	78.167 401	0

				v_4				
a"	47.507 936	a	47.507 799	137	b"	42.942 662	b	42.942 799
d"	47.379 737	d	47.379 600	137	e"	69.489 203	e	69.489 340
f"	25.841 713	f	25.841 576	137	g"	25.006 150	g	25.006 287
i"	55.535 302	i	55.535 165	137	j"	46.297 297	j	46.297 434

Log sin a"	9.867 6861	δ 6 9437	Log sin b"	9.833 3169	δ 8 1449
Log sin d"	9.866 7938	δ 6 9750	Log sin e"	9.971 5570	δ 2 8356
Log sin f"	9.639 3734	δ 15 6506	Log sin g"	9.626 0484	δ 16 2502
Log sin i"	<u>9.916 1775</u>	δ 5 2026	Log sin j"	<u>9.859 0990</u>	δ 7 2442
sum 1	9.290 0309		sum 2	9.290 0213	

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	$\bar{t}_{AB}$ = 176.853 693		$\bar{T}_{AB}$ =	176.847 414
$\bar{t}_{AB}$ + j	= $\bar{t}_{AC}$ = 223.151 127;	$\bar{t}_{AC} + (T-t)_{AC}$	= $\bar{T}_{AC}$ =	
$\bar{t}_{AC}$ + 180	= $\bar{t}_{CA}$ = 43.151 127;	$\bar{t}_{CA} + (T-t)_{CA}$	= $\bar{T}_{CA}$ =	
$\bar{t}_{AC}$ + a	= $\bar{t}_{AD}$ = 270.658 926;	$\bar{t}_{AD} + (T-t)_{AD}$	= $\bar{T}_{AD}$ =	
$\bar{t}_{AD}$ + 180	= $\bar{t}_{DA}$ = 90.658 926;	$\bar{t}_{DA} + (T-t)_{DA}$	= $\bar{T}_{DA}$ =	
$\bar{t}_{DA}$ + b	= $\bar{t}_{DC}$ = 133.601 725;	$\bar{t}_{DC} + (T-t)_{DC}$	= $\bar{T}_{DC}$ =	
$\bar{t}_{DC}$ + 180	= $\bar{t}_{CD}$ = 313.601 725;	$\bar{t}_{CD} + (T-t)_{CD}$	= $\bar{T}_{CD}$ =	
$\bar{t}_{DC}$ + d	= $\bar{t}_{DE}$ = 180.981 325;	$\bar{t}_{DE} + (T-t)_{DE}$	= $\bar{T}_{DE}$ =	180.975 936
$\bar{t}_{DE}$ + 180	= $\bar{t}_{ED}$ = 0.981 325;	$\bar{t}_{ED} + (T-t)_{ED}$	= $\bar{T}_{ED}$ =	0.986 714
$\bar{t}_{ED}$ + e	= $\bar{t}_{EC}$ = 70.470 665;	$\bar{t}_{EC} + (T-t)_{EC}$	= $\bar{T}_{EC}$ =	
$\bar{t}_{EC}$ + 180	= $\bar{t}_{CE}$ = 250.470 665;	$\bar{t}_{CE} + (T-t)_{CE}$	= $\bar{T}_{CE}$ =	
$\bar{t}_{EC}$ + f	= $\bar{t}_{EB}$ = 96.312 241;	$\bar{t}_{EB} + (T-t)_{EB}$	= $\bar{T}_{EB}$ =	
$\bar{t}_{EB}$ + 180	= $\bar{t}_{BE}$ = 276.312 241;	$\bar{t}_{BE} + (T-t)_{BE}$	= $\bar{T}_{BE}$ =	
$\bar{t}_{BE}$ + g	= $\bar{t}_{BC}$ = 301.318 528;	$\bar{t}_{BC} + (T-t)_{BC}$	= $\bar{T}_{BC}$ =	
$\bar{t}_{BC}$ + 180	= $\bar{t}_{CB}$ = 121.318 528;	$\bar{t}_{CB} + (T-t)_{CB}$	= $\bar{T}_{CB}$ =	
$\bar{t}_{BC}$ + i	= $\bar{t}_{BA}$ = 356.853 693;		$\bar{T}_{BA}$ =	356.859 974

$\tan \bar{t}_{AC}$	+0.9374 58592	$\tan \bar{t}_{AD}$	-86.9494 5040
$\tan \bar{t}_{BC}$	-1.6435 13571	$\tan \bar{t}_{CD}$	- 1.0500 40144
diff.	+2.5809 72163	diff.	-85.8994 1026
$\tan \bar{t}_{DE}$	+0.0171 29027	$\tan \bar{t}_{EB}$	- 9.0401 78673
$\tan \bar{t}_{CE}$	+2.8193 24651	$\tan \bar{t}_{CB}$	- 1.6435 13571
diff.	-2.8021 95624	diff.	- 7.3966 65102

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$$\frac{(\eta_B-\eta_A) - \tan \bar{t}_{BC}(\xi_B-\xi_A)}{\tan \bar{t}_{AC} - \tan \bar{t}_{BC}} = \xi_C-\xi_A = -\ 5\ 957.05; \quad \bar{\xi}_C = 2951\ 740.37$$

$$\tan \bar{t}_{AC}(\xi_C-\xi_A) = \eta_C-\eta_A = -\ 5\ 584.48; \quad \bar{\eta}_C = 917\ 511.34$$

$$\frac{(\eta_C-\eta_A) - \tan \bar{t}_{CD}(\xi_C-\xi_A)}{\tan \bar{t}_{AD} - \tan \bar{t}_{CD}} = \xi_D-\xi_A = +\ 137.83; \quad \bar{\xi}_D = 2957\ 835.25$$

$$\tan \bar{t}_{AD}(\xi_D-\xi_A) = \eta_D-\eta_A = -11\ 984.35; \quad \bar{\eta}_D = 911\ 111.47$$

$$\frac{(\eta_C-\eta_D) - \tan \bar{t}_{CE}(\xi_C-\xi_D)}{\tan \bar{t}_{DE} - \tan \bar{t}_{CE}} = \xi_E-\xi_D = -\ 8\ 416.01; \quad \bar{\xi}_E = 2949\ 419.24$$

$$\tan \bar{t}_{DE}(\xi_E-\xi_D) = \eta_E-\eta_D = -\ 144.16; \quad \bar{\eta}_E = 910\ 967.31$$

$$\frac{(\eta_C-\eta_E) - \tan \bar{t}_{CB}(\xi_C-\xi_E)}{\tan \bar{t}_{EB} - \tan \bar{t}_{CB}} = \xi_B-\xi_E = -\ 1\ 400.47; \quad \xi_B' = 2948\ 018.77$$

$$\tan \bar{t}_{EB}(\xi_B-\xi_E) = \eta_B-\eta_E = +12\ 660.54; \quad \eta_B' = 923\ 627.85$$

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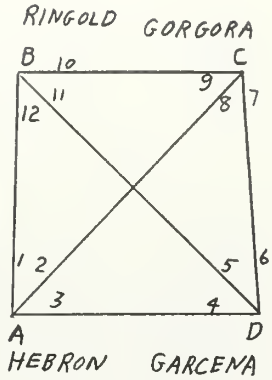
Stations: A. Corpus; B. Monument; C. Grande; D. Hebron;
E. Ringold

Datum: North American 1913; Ellipsoid: Clarke 1866; λ_o 108°W



ξ_B	2949 419.24	η_B	910 967.31
ξ_A	2957 835.25	η_A	911 111.47
$\xi_B - \xi_A$	− 8 416.01	$\eta_B - \eta_A$	− 144.16
$\overline{T}_{AB}$	180.975 936	$\overline{T}_{BA}$	0.986 714
−1 + 2	71.553 875	+11 − 12	−29.415 125
T'_{AC}	252.529 811	T'_{BD}	331.571 589
−2 + 3	11.725 447	+10 − 11	−41.328 886
T'_{AD}	264.255 258	T'_{BC}	290.242 703

$\tan T'_{AC}$	+3.1773 58316	$(T-t)_{AB}$	−.005 3894	$\tan T'_{AD}$	+9.9401 59622
$\tan T'_{BC}$	−2.7116 82080	$(T-t)_{BA}$	+.005 3892	$\tan T'_{BD}$	−0.5413 38989
diff	+5.8890 40396			diff	+10.4814 9861

$$\xi'_i - \xi_A = \frac{(\eta_B - \eta_A) - \tan T'_{Bi}(\xi_B - \xi_A)}{\tan T'_{Ai} - \tan T'_{Bi}}; \quad \eta'_i = \eta_A + \tan T'_{Ai}(\xi'_i - \xi_A).$$

$\xi'_C - \xi_A = -3\,899.74$; $\xi'_C = 2953\,935.51$; $\eta'_C = 898\,720.61$.

$\xi'_D - \xi_A = -\;448.42$; $\xi'_D = 2957\,386.83$; $\eta'_D = 906\,654.14$.

$\xi'_r = \xi'(1 - H_2\eta^2)$ $\quad \xi'_{rA} = 2927\,510 \quad \xi'_{rC} = 2924\,469$

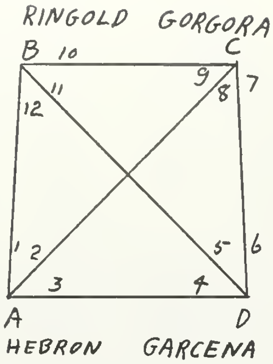
$\quad \xi'_{rB} = 2919\,190 \quad \xi'_{rD} = 2927\,363$

$$\xi_{3ij} = \frac{1}{3}(2\xi'_{ri} + \xi'_{rj}); \qquad \eta_{3ij} = \frac{1}{3}(2\eta'_i + \eta'_j)$$

	ξ_3	η_3		ξ_3	η_3
AC	2926 496	906 981.18	; BC	2920 950	906 885.08
AD	2927 461	909 625.69	; BD	2921 914	909 529.59

$$(T-t) = I_1(\xi_j - \xi_i)\eta_3 + I_2(\eta_j - \eta_i)\eta_3^2 - I_3(\xi_j - \xi_i)\eta_3^3 - I_4(\eta_j - \eta_i)\eta_3^4$$

Line	AC	AD	BC	BD
$\xi_j - \xi_i$	− 3 899.74	− 448.42	+ 4 516.27	+ 7 967.59
$\eta_j - \eta_i$	−12 390.86	− 4 457.33	−12 246.70	− 4 313.17
$(T-t)_1$	−.002 5030	−.000 2886	+.002 8984	+.005 1283
$(T-t)_2$	− 61	− 22	− 61	− 22
$(T-t)_3$	+ 168	+ 19	− 194	− 346
$(T-t)_4$	negligible	negligible	negligible	negligible
$(T-t)$	−.002 4923	−.000 2889	+.002 8729	+.005 0915



ξ_B	2949 419.24	η_B	910 967.31
ξ_A	2957 835.25	η_A	911 111.47
$\xi_B - \xi_A$	- 8 416.01	$\eta_B - \eta_A$	- 144.16
$\bar{t}_{AB}$	180.981 325	$\bar{t}_{BA}$	0.981 325
-1' + 2'	71.550 978	+11' - 12'	-29.414 827
t''_{AC}	252.532 303	t''_{BD}	331.566 498
-2' + 3'	11.723 244	+10' - 11'	-41.326 667
t''_{AD}	264.255 547	t''_{BC}	290.239 831

tan t''_{AC}	+3.1778 40974	$(T-t)_{AB}$	-.005 3894	tan t''_{AD}	+9.9406 63068
tan t''_{BC}	-2.7121 00849	$(T-t)_{BA}$	+.005 3892	tan t''_{BD}	-0.5414 53888
diff.	+5.8899 41823			diff.	+10.4821 1696

$-1' + 2' = -1 + 2 + (T-t)_{AB} - (T-t)_{AC}$ etc.

$\xi''_C - \xi_A =$ -3 899.74; $\xi''_C =$ 2953 935.51; $\eta''_C =$ 898 718.72.

$\xi''_D - \xi_A =$ - 448.48; $\xi''_D =$ 2957 386.77; $\eta''_D =$ 906 653.26.

$\xi''_r = \xi''(1 - H_2\eta^2)$

ξ''_{rA}	2927 510	$\xi''_{rC} =$	2924 469
ξ''_{rB}	2919 190	$\xi''_{rD} =$	2927 363

	ξ_3	η_3		ξ_3	η_3
AC	2926 496	906 980.55	; CA	2925 483	902 849.64
AD	2927 461	909 625.40	; DA	2927 412	908 139.33
BC	2920 950	906 884.45	; CB	2922 709	902 801.58
BD	2921 914	909 529.29	; DB	2924 639	908 091.28
CD	2925 434	901 363.57	; DC	2926 398	904 008.41

Line	AC	AD	BC	BD	CD
$\xi_j - \xi_i$	- 3 899.74	- 448.48	+ 4 516.27	+ 7 967.53	+ 3 451.26
$\eta_j - \eta_i$	-12 392.75	- 4 458.21	-12 248.59	- 4 314.05	+ 7 934.54
$(T-t)_1$	-.002 5030	-.000 2887	+.002 8984	+.005 1282	+.002 2014
$(T-t)_2$	- 61	- 22	- 61	- 22	+ 39
$(T-t)_3$	+ 168	+ 19	- 194	- 346	- 146
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	-.002 4923	-.000 2890	+.002 8729	+.005 0914	+.002 1907

Line	CA	DA	CB	DB	DC
$\xi_j-\xi_i$	+ 3 899.74	+ 448.48	- 4 516.27	- 7 967.53	- 3 451.26
$\eta_j-\eta_i$	+12 392.75	+ 4 458.21	+12 248.59	+ 4 314.05	- 7 934.54
$(T-t)_1$	+.002 4916	+.000 2882	-.002 8854	-.005 1201	-.002 2079
$(T-t)_2$	+ 61	+ 22	+ 60	+ 21	- 39
$(T-t)_3$	- 165	- 19	+ 192	+ 344	+ 147
$(T-t)_4$	negligible	negligible	negligible	negligible	negligible
$(T-t)$	+.002 4812	+.000 2885	-.002 8602	-.005 0836	-.002 1971

$$-t'_{ij} + t'_{ik} = -ij + ik + (T-t)_{ij} - (T-t)_{ik}$$

$-t'_{AC} + t'_{AD} = a' = 11.723\ 244$ $\qquad -t'_{CA} + t'_{CB} = e' = 37.707\ 558$

$-t'_{DA} + t'_{DB} = b' = 67.310\ 980$ $\qquad -t'_{BC} + t'_{BD} = f' = 41.326\ 668$

$-t'_{DB} + t'_{DC} = c' = 94.926\ 412$ $\qquad -t'_{BD} + t'_{BA} = g' = 29.414\ 827$

$-t'_{CD} + t'_{CA} = d' = 6.039\ 734$ $\qquad -t'_{AB} + t'_{AC} = h' = 71.550\ 978$

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		v_1	v_2	v_3	
					$v_1 = -(a'+b'+\dots h')/8$
a'	11.723 244	-50	+1		$v_2 = (a'+b'-(e'+f'))/4$
b'	67.310 980	-50	0		subtract from a' and b',
c'	94.926 412	-50		-85	add to e' and f'.
d'	6.039 734	-51		-85	$v_3 = (c'+d'-(g'+h'))/4$
e'	37.707 558	-50	-1		subtract from c' and d',
f'	41.326 668	-50	0		add to g' and h'.
g'	29.414 827	-50		+85	$v_4 = (\text{sum 1} - \text{sum 2})/\Sigma\delta$,
h'	71.550 978	-50		+85	subtract from a", c", e",
					g", add to other angles.

				v_4				
a"	11.723 195	a	11.723 148	41	b"	67.310 930	b	67.310 977
c"	94.926 277	c	94.926 230	41	d"	6.039 598	d	6.039 645
e"	37.707 507	e	37.707 460	41	f"	41.326 618	f	41.326 665
g"	29.414 862	g	29.414 815	41	h"	71.551 013	h	71.551 060

Log sin a"	9.307 8888	δ 36 5273	Log sin b"	9.965 0190	δ 3 1690
Log sin c"	9.998 3927	δ - 6535	Log sin d"	9.022 0808	δ 71 6413
Log sin e"	9.786 4893	δ 9 8046	Log sin f"	9.819 7746	δ 8 6198
Log sin g"	9.691 1962	δ 13 4439	Log sin h"	9.977 0858	δ 2 5286
sum 1	8.783 9670		sum 2	8.783 9602	

	$\bar{t}_{AB} = 180.981\ 325$;		$\bar{T}_{AB} = 180.975\ 936$
$\bar{t}_{AB} + h$	$= \bar{t}_{AC} = 252.532\ 385$;	$\bar{t}_{AC} + (T-t)_{AC}$	$= \bar{T}_{AC} =$
$\bar{t}_{AC} + a$	$= \bar{t}_{AD} = 264.255\ 533$;	$\bar{t}_{AD} + (T-t)_{AD}$	$= \bar{T}_{AD} =$
$\bar{t}_{AD} + 180$	$= \bar{t}_{DA} = 84.255\ 533$;	$\bar{t}_{DA} + (T-t)_{DA}$	$= \bar{T}_{DA} =$
$\bar{t}_{DA} + b$	$= \bar{t}_{DB} = 151.566\ 510$;	$\bar{t}_{DB} + (T-t)_{DB}$	$= \bar{T}_{DB} =$
$\bar{t}_{DB} + c$	$= \bar{t}_{DC} = 246.492\ 740$;	$\bar{t}_{DC} + (T-t)_{DC}$	$= \bar{T}_{DC} = 246.490\ 543$
$\bar{t}_{DC} + 180$	$= \bar{t}_{CD} = 66.492\ 740$;	$\bar{t}_{CD} + (T-t)_{CD}$	$= \bar{T}_{CD} = 66.494\ 931$
$\bar{t}_{CD} + d$	$= \bar{t}_{CA} = 72.532\ 385$;	$\bar{t}_{CA} + (T-t)_{CA}$	$= \bar{T}_{CA} =$
$\bar{t}_{CA} + e$	$= \bar{t}_{CB} = 110.239\ 845$;	$\bar{t}_{CB} + (T-t)_{CB}$	$= \bar{T}_{CB} =$
$\bar{t}_{CB} + 180$	$= \bar{t}_{BC} = 290.239\ 845$;	$\bar{t}_{BC} + (T-t)_{BC}$	$= \bar{T}_{BC} =$
$\bar{t}_{BC} + f$	$= \bar{t}_{BD} = 331.566\ 510$;	$\bar{t}_{BD} + (T-t)_{BD}$	$= \bar{T}_{BD} =$
$\bar{t}_{BD} + g$	$= \bar{t}_{BA} = 0.981\ 325$;		$\bar{T}_{BA} = 0.986\ 714$

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tan $\bar{t}_{AD}$	+9.9406 38690	tan $\bar{t}_{DC}$	+2.2990 45859	tan $\bar{t}_{CB}$	-2.7120 98808
tan $\bar{t}_{BD}$	<u>-0.5414 53617</u>	tan $\bar{t}_{AC}$	<u>+3.1778 56859</u>	tan $\bar{t}_{DB}$	<u>-0.5414 53617</u>
diff.	+10.4820 9231	diff.	-0.8788 11000	diff.	-2.1706 45191

$$\frac{(\eta_B-\eta_A) - \tan \bar{t}_{BD}(\xi_B-\xi_A)}{\tan \bar{t}_{AD} - \tan \bar{t}_{BD}} = \xi_D-\xi_A = - \quad 448.48; \quad \bar{\xi}_D = 2957\ 386.77$$

$$\tan \bar{t}_{AD}(\xi_D-\xi_A) = \eta_D-\eta_A = -\ 4\ 458.21; \quad \bar{\eta}_D = 906\ 653.26$$

$$\frac{\tan \bar{t}_{AC}(\xi_D-\xi_A) - (\eta_D-\eta_A)}{\tan \bar{t}_{AC} - \tan \bar{t}_{DC}} = \xi_C-\xi_D = -\ 3\ 451.25; \quad \bar{\xi}_C = 2953\ 935.52$$

$$\tan \bar{t}_{DC}(\xi_C-\xi_D) = \eta_C-\eta_D = -\ 7\ 934.58; \quad \bar{\eta}_C = 898\ 718.68$$

$$\frac{\tan \bar{t}_{DB}(\xi_C-\xi_D) - (\eta_C-\eta_D)}{\tan \bar{t}_{CB} - \tan \bar{t}_{DB}} = \xi_B-\xi_C = -\ 4\ 516.29; \quad \xi'_B = 2949\ 419.23$$

$$\tan \bar{t}_{CB}(\xi_B-\xi_C) = \eta_B-\eta_C = +12\ 248.64; \quad \eta'_B = 910\ 967.32$$

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Stations: A. Hebron; B. Ringold; C. Gorgora; D. Garcena.

Datum: North American 1913; Ellipsoid: Clarke 1866; λ_o 108°W

v	dz_1	dz_2	dz_3	dz_4	dz_5	dz_6	dz_7	dz_8	dz_9	dz_{10}	dz_{11}	dz_{12}	dz_{13}
1	-1												
2	-1												
3	-1												
4		-1											
5		-1											
6		-1											
7			-1										
8			-1										
9			-1										
10			-1										
11			-1										
12				-1									
13				-1									
14				-1									
15				-1									
16				-1									
17					-1								
18					-1								
19					-1								
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21					-1								
22						-1							
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25						-1							
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38									-1				
39									-1				
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52											-1		
53												-1	
54												-1	
55												-1	
56													-1
57													-1
58													-1

v	$d\eta_3$	$d\xi_3$	$d\eta_4$	$d\xi_4$	$d\eta_5$	$d\xi_5$	$d\eta_6$	$d\xi_6$	$d\eta_7$	$d\xi_7$	$d\eta_8$	$d\xi_8$	$d\eta_9$	$d\xi_9$	$d\eta_{10}$	$d\xi_{10}$	$d\eta_{11}$	$d\xi_{11}$	l_1	l_2
1	+2901.9	+2118.7																	-67.	-47.
2			+2309.1	+4483.5															+20.	+73.
3																			0	-25.
4			-1665.4	+4393.6															-4.	+6.
5	+1527.3	+4873.6																	-49.	-23.
6																			0	+18.
7	+1527.3	+4873.6																	+194.	+192.
8	+2901.9	+2118.7																	+80.	+115.
9	+7402.3	-672.4	-7402.3	+672.4															-47.	-64.
10	+889.9	-3870.1			-889.9	+3870.1													-84.	-93.
11	-1992.8	-3227.0					+1992.8	+3227.0											-143.	-151.
12			+2309.1	+4483.5															-131.	-57.
13			-1665.4	+4393.6															-114.	-126.
14	+7402.3	-672.4	-7402.3	+672.4															+84.	+73.
15			-2284.7	-1778.6			+2284.7	+1778.6											+114.	+98.
16			-1283.6	-3860.5	+1283.6	+3860.5													+49.	+14.
17			-1283.6	-3860.5	+1283.6	+3860.5													+41.	+43.
18	+889.9	-3870.1			-889.9	+3870.1													-31.	+3.
19					-5069.7	+548.2	+5069.7	-548.2											-64.	-19.
20					-3942.8	-2508.5			+3942.8	+2508.5									-27.	-18.
21					-1030.0	-9357.8					+1030.0	+9357.8							+81.	-8.
22	-1992.8	-3227.0					+1992.8	+3227.0											+117.	+118.
23			-2284.7	-1778.6			+2284.7	+1778.6											+48.	+35.
24					-5069.7	+548.2	+5069.7	-548.2											+92.	+103.
25							+3692.0	-2550.6			-3692.0	+2550.6							-88.	-83.
26							+771.9	-7273.7	-771.9	+7273.7									-170.	-172.
27							+771.9	-7273.7	-771.9	+7273.7									+213.	+222.
28					-3942.8	-2508.5			+3942.8	+2508.5									-12.	-26.
29									+5902.0	+324.4	-5902.0	-324.4							-23.	+4.
30									+5119.2	-4799.1			-5119.2	+4799.1					+72.	+45.
31									-55.0	-4780.3							+55.0	+4780.3	-252.	-246.
32					-1030.0	-9357.8					+1030.0	+9357.8							-104.	-208.
33							+3692.0	-2550.6			-3692.0	+2550.6							-138.	-114.
34									+5902.0	+324.4	-5902.0	-324.4							+15.	+50.
35											-2222.9	-2834.3					+2222.9	+2834.3	+67.	+87.
36											-4159.7	-6836.5	+4159.7	+6836.5					+195.	+215.
37											-494.6	-4470.8			+494.6	+4470.8			-37.	-32.
38									+5119.2	-4799.1			-5119.2	+4799.1					0	-35.
39											-4159.7	-6836.5	+4159.7	+6836.5					0	+4.
40													+2758.5	-7777.0	-2758.5	+7777.0			0	+34.
41													-4471.0	-4694.7			+4471.0	+4694.7	0	-2.
42											-494.6	-4470.8			+494.6	+4470.8			+22.	-80.
43													+2758.5	-7777.0	-2758.5	+7777.0			+64.	+7.
44															-6806.0	+116.6	+6806.0	-116.6	-38.	-120.
45															-5560.8	-3010.9			-130.4	-229.
46															-1518.4	-4117.8			+557.6	+421.
47									-55.0	-4780.3							+55.0	+4780.3	+139.	-389.
48											-2222.9	-2834.3					+2222.9	+2834.3	-51.	-573.
49													-4471.0	-4694.7			+4471.0	+4694.7	+90.	-438.
50															-6806.0	+116.6	+6806.0	-116.6	-82.	-599.
51																	+1323.7	-4206.7	+1053.1	+539.
52																	+1279.4	-12722.8	+1940.4	+1460.
53																	+1279.4	-12722.8	+2099.4	+1334.
54															-5560.8	-3010.9			-44.4	-863.
55																			+340.1	-472.
56															-1518.4	-4117.8			+485.6	-247.
57																	+1323.7	-4206.7	+1056.1	+381.
58																			+554.1	-134.

TABLE XV - Error Equations

TABLE XVI

Symbolic

Error Equations

v	$d\eta_3$	$d\xi_3$	$d\eta_4$	$d\xi_4$	$d\eta_5$	$d\xi_5$	$d\eta_6$	$d\xi_6$	$d\eta_7$	$d\xi_7$	$d\eta_8$	$d\xi_8$	$d\eta_9$	$d\xi_9$	$d\eta_{10}$	$d\xi_{10}$	$d\eta_{11}$	$d\xi_{11}$	l_1	sum_1	l_2	sum_2
1	+2901.9	+2118.7																	- 67.	+ 4953.7	- 47.	+ 4973.7
2			+2309.1	+4483.5															+ 20.	+ 6812.6	- 73.	+ 6865.6
3																			0	0	- 25.	- 25.
3s	+1675.4i	+1223.3i	+1333.2i	+2588.5i															- 27.1i	+ 6793.3i	+ .6i	+ 6821.0i
4			-1665.4	+4393.6															- 4.	+ 2724.2	+ 6.	+ 2734.2
5	+1527.3	+4873.6																	- 49.	+ 6351.9	- 23.	+ 6377.9
6																			0	0	+ 18.	+ 18.
6s	+ 881.8i	+2813.8i	- 961.5i	+2536.7i															- 30.6i	+ 5240.1i	+ .6i	+ 5271.2i
7	+1527.3	+4873.6																	+ 194.	+ 6594.9	+ 192.	+ 6592.9
8	+2901.9	+2118.7																	+ 80.	+ 5100.7	+ 115.	+ 5135.7
9	+7402.3	- 672.4	-7402.3	+ 672.4															- 47.	- 47.	- 64.	- 64.
10	+ 889.9	-3870.1			- 889.9	+3870.1													- 84.	- 84.	- 93.	- 93.
11	-1992.8	-3227.0					+1992.8	+3227.0											- 143.	- 143.	- 151.	- 151.
11s	+4798.0i	- 347.5i	-3310.4i	+ 300.7i	- 398.0i	+1730.8i	+ 891.2i	+1443.1i											0	+ 4506.5i	- .4i	+ 4506.1i
12			+2309.1	+4483.5															- 131.	+ 6661.6	- 57.	+ 6735.6
13			-1665.4	+4393.6															- 114.	+ 2614.2	- 126.	+ 2602.2
14	+7402.3	- 672.4	-7402.3	+ 672.4															+ 84.	+ 84.	+ 73.	+ 73.
15			-2284.7	-1778.6			+2284.7	+1778.6											+ 114.	+ 114.	+ 98.	+ 98.
16			-1283.6	-3860.5	+1283.6	+3860.5													+ 49.	+ 49.	+ 14.	+ 14.
16s	+3310.4i	- 300.7i	-4618.4i	+1748.7i	+ 574.1i	+1726.5i	+1021.7i	+ 795.4i											+ .9i	+ 4258.7i	+ .9i	+ 4258.7i
17			-1283.6	-3860.5	+1283.6	+3860.5													+ 41.	+ 41.	+ 43.	+ 43.
18	+ 889.9	-3870.1			- 889.9	+3870.1													- 31.	- 31.	+ 3.	+ 3.
19					-5069.7	+ 548.2	+5069.7	- 548.2											- 64.	- 64.	- 19.	- 19.
20					-3942.8	-2508.5			+3942.8	+2508.5									- 27.	- 27.	- 18.	- 18.
21					-1030.0	-9357.8					+1030.0	+9357.8							+ 81.	+ 81.	- 8.	- 8.
21s	+ 398.0i	-1730.8i	- 574.1i	-1726.5i	-4315.0i	-1604.4i	+2267.2i	- 245.1i	+1763.3i	+1121.8i	+ 460.6i	+4184.9i							0	0	+ .4i	+ .4i
22	-1992.8	-3227.0					+1992.8	+3227.0											+ 117.	+ 117.	+ 118.	+ 118.
23			-2284.7	-1778.6			+2284.7	+1778.6											+ 48.	+ 48.	+ 35.	+ 35.
24					-5069.7	+ 548.2	+5069.7	- 548.2											+ 92.	+ 92.	+ 103.	+ 103.
25							+3692.0	-2550.6			-3692.0	+2550.6							- 88.	- 88.	- 83.	- 83.
26							+ 771.9	-7273.7	- 771.9	+7273.7									- 170.	- 170.	- 172.	- 172.
26s	- 891.2i	-1443.1i	-1021.7i	- 795.4i	-2267.2i	+ 245.1i	+6176.5i	-2400.1i	- 345.2i	+3252.9i	-1651.1i	+1140.6i							- .4i	- .4i	+ .4i	+ .4i
27							+ 771.9	-7273.7	- 771.9	+7273.7									+ 213.	+ 213.	+ 222.	+ 222.
28					-3942.8	-2508.5			+3942.8	+2508.5									- 12.	- 12.	- 26.	- 26.
29									+5902.0	+ 324.4	-5902.0	- 324.4							- 23.	- 23.	+ 4.	+ 4.
30									+5119.2	-4799.1			-5119.2	+4799.1					+ 72.	+ 72.	+ 45.	+ 45.
31									- 55.0	-4780.3							+ 55.0	+ 4780.3	- 252.	- 252.	- 246.	- 246.
31s					-1763.3i	-1121.8i	+ 345.2i	-3252.9i	+6322.3i	+ 235.8i	-2639.4i	- 145.1i	-2289.4i	+2146.2i			+ 24.6i	+ 2137.8i	- .9i	- .9i	- .4i	- .4i
32					-1030.0	-9357.8					+1030.0	+9357.8							- 104.	- 104.	- 208.	- 208.
33							+3692.0	-2550.6			-3692.0	+2550.6							- 138.	- 138.	- 114.	- 114.
34									+5902.0	+ 324.4	-5902.0	- 324.4							+ 15.	+ 15.	+ 50.	+ 50.
35											-2222.9	-2834.3					+2222.9	+ 2834.3	+ 67.	+ 67.	+ 87.	+ 87.
36											-4159.7	-6836.5	+4159.7	+6836.5					+ 195.	+ 195.	+ 215.	+ 215.
37											- 494.6	-4470.8			+ 494.6	+4470.8			- 37.	- 37.	- 32.	- 32.
37s					- 420.5i	-3820.3i	+1507.3i	-1041.3i	+2409.5i	+ 132.4i	-6303.8i	-1044.2i	+1698.2i	+2791.0i	+ 201.9i	+1825.2i	+ 907.5i	+ 1157.1i	- .8i	- .8i	- .8i	- .8i
38									+5119.2	-4799.1			-5119.2	+4799.1					0	0	- 35.	- 35.
39											-4159.7	-6836.5	+4159.7	+6836.5					0	0	+ 4.	+ 4.
40													+2758.5	-7777.0	-2758.5	+7777.0			0	0	+ 34.	+ 34.
41													-4471.0	-4694.7			+4471.0	+ 4694.7	0	0	- 2.	- 2.
41s									+2559.6i	-2399.5i	-2079.8i	-3418.2i	-1336.1i	- 418.1i	-1379.2i	+3888.5i	+2235.5i	+ 2347.4i	0	0	+ .5i	+ .5i
42											- 494.6	-4470.8			+ 494.6	+4470.8			+ 22.	+ 22.	- 80.	- 80.
43													+2758.5	-7777.0	-2758.5	+7777.0			+ 64.	+ 64.	+ 7.	+ 7.
44															-6806.0	+ 116.6	+6806.0	- 116.6	- 38.	- 38.	- 120.	- 120.
45															-5560.8	-3010.9			- 130.4	- 8702.1	- 229.	- 8800.7
46															-1518.4	-4117.8			+ 557.6	- 5078.5	+ 421.	- 5215.2
46s											- 221.2i	-1999.4i	+1233.6i	-3478.0i	-7222.0i	+2341.5i	+3043.7i	- 52.1i	+ 212.5i	- 6141.4i	- .4i	- 6354.4i
47									- 55.0	-4780.3							+ 55.0	+ 4780.3	+ 139.	+ 139.	- 389.	- 389.
48											-2222.9	-2834.3					+2222.9	+ 2834.3	- 51.	- 51.	- 573.	- 573.
49													-4471.0	-4694.7			+4471.0	+ 4694.7	+ 90.	+ 90.	- 438.	- 438.
50															-6806.0	+ 116.6	+6806.0	- 116.6	- 82.	- 82.	- 599.	- 599.
51																	+1323.7	- 4206.7	+1053.1	- 1829.9	+ 539.	- 2344.1
52																	+1279.4	-12722.8	+1940.4	- 9503.0	+1460.	- 9983.4
52s									- 22.4i	-1951.5i	- 907.5i	-1157.1i	-1825.3i	-1916.6i	-2778.5i	+ 47.6i	+6596.4i	- 1933.8i	+1261.3i	- 4587.5i	0	- 5848.8i
53																	+1279.4	-12722.8	+2099.4	- 9344.0	+1334.	-10109.4
54															-5560.8	-3010.9			- 44.4	- 8616.1	- 863.	- 9434.7
55																			+ 340.1	+ 340.1	- 472.	- 472.
55s															-3210.5i	-1738.3i	+ 738.7i	- 7345.5i	+1382.8i	-10172.9i	- .6i	-11556.3i
56															-1518.4	-4117.8			+ 485.6	- 5150.5	- 247.	- 5883.2
57																	+1323.7	- 4206.7	+1056.1	- 1826.9	+ 381.	- 2502.1
58																			+ 554.1	+ 554.1	- 134.	- 134.
58s															- 876.6i	-2377.4i	+ 764.2i	- 2428.8i	+1210.0i	- 3708.5i	0	- 4918.6i

TABLE XVI

Symbolic error equations

$d\eta_3$	$d\xi_3$	$d\eta_4$	$d\xi_4$	$d\eta_5$	$d\xi_5$	$d\eta_6$	$d\xi_6$	$d\eta_7$	$d\xi_7$	$d\eta_8$	$d\xi_8$	$d\eta_9$	$d\xi_9$	$d\eta_{10}$	$d\xi_{10}$	$d\eta_{11}$	$d\xi_{11}$	l_1	sum_1	l_2	sum_2
+102106097	+ 20738016	- 80484226	- 987857	- 1878068	- 6274348	- 10998999	- 24460470	- 1009387	+ 2452582	- 1654834	- 648966	0	0	0	0	0	0	+ 551627	- 2548833	+ 505665	- 2594795
	+ 93465060	+ 6021595	- 14922869	- 3817702	- 31257740	+ 592864	- 23973794	+ 2553626	+ 6636016	- 1585558	+ 8889217	0	0	0	0	0	0	+ 1357219	+ 64695950	+ 1415663	+ 64754394
		+103171103	+ 18420496	- 6755351	+ 3121460	+ 4841829	- 2269319	+ 659529	+ 3967630	- 1422588	+ 3567857	0	0	0	0	0	0	- 808893	+ 52031123	- 203704	+ 52636312
			+ 95952459	- 20287849	- 34881048	- 818756	- 9616529	+ 2769659	+ 4524292	- 518091	+ 8132501	0	0	0	0	0	0	- 1480881	+ 46285526	- 909585	+ 46856821
				+ 61961305	+ 26268577	- 26605859	- 6997181	- 12103798	- 7093661	- 11182254	+ 672633	- 3322719	+ 4957878	+ 84896	+ 767479	+ 424942	+ 4256017	+ 249928	- 400788	+ 124486	- 526230
					+223621393	+ 10520246	- 11904100	- 569872	- 10812386	- 45176318	-172853142	+ 3919189	+ 13070049	+ 771316	+ 6972851	+ 3494473	+ 6818684	+ 225239	- 14925475	+ 2045659	- 13105055
						+ 50720183	+ 1342556	- 8871228	- 11687523	- 7695446	+ 3923900	- 1769318	- 4947531	- 304313	- 2751054	- 1376307	- 2481950	- 337558	- 8704265	- 27685	- 8394393
							+126376771	+ 33906916	- 96825616	- 166314	- 10806373	- 5678886	+ 9887513	+ 210231	+ 1900529	+ 1024917	+ 8158841	+ 447005	- 9443304	+ 221613	- 9668696
								+ 98810357	- 32794619	- 33869364	+ 1341694	- 38651700	+ 29868426	+ 2981470	- 14349817	- 7922027	- 22881136	+ 176400	+ 45130	+ 195616	+ 64346
									+188893192	- 4279594	- 18903465	+ 42682013	- 51681342	- 8758610	+ 9181735	+ 17585691	- 44500341	+ 2870182	- 8543826	+ 3257754	- 8156253
										+ 89186230	+ 62559084	- 34105645	- 36994280	- 6203776	+ 15732156	+ 7211884	+ 3451712	+ 1201996	- 5510999	+ 406038	- 6306957
											+300927745	- 59646074	-100849543	- 26580699	- 20042486	+ 9710311	- 8865872	- 216621	- 19688299	- 2162739	- 21634417
												+127452807	+ 7222082	- 13566115	+ 42199305	- 30192084	- 39380086	+ 2256073	- 581157	+ 2941689	+ 104459
													+276240736	+ 11321552	-116197373	- 20402190	- 54804650	+ 3919080	- 29389592	+ 3293887	- 30014785
														+101907269	+ 20292486	- 46390363	- 26870759	+ 10560688	+ 19455272	+ 10528912	+ 19423497
															+180401014	- 13102039	- 29595964	+ 1371659	+ 82780481	+ 2305300	+ 83714122
																+ 89559693	+ 23674187	- 3336660	- 29964429	- 3182068	+ 30119021
																	+389991549	- 44779124	+162191109	- 45816018	+161154215
																		+ 6811674	- 18960966	+ 7096453	- 17963064

TABLE XVII - Normal Equations

TABLE XVII

Normal Equations

	$d\eta_3$	$d\xi_3$	$d\eta_4$	$d\xi_4$	$d\eta_5$	$d\xi_5$	$d\eta_6$	$d\xi_6$	$d\eta_7$	$d\xi_7$	$d\eta_8$	$d\xi_8$	$d\eta_9$	$d\xi_9$	$d\eta_{10}$	$d\xi_{10}$	$d\eta_{11}$	$d\xi_{11}$	l_1	sum_1	l_2	sum_2
-1.	+102106097	+ 20738016	- 80484226	- 987857	- 1878068	- 6274348	- 10998999	- 24460470	- 1009387	+ 2452582	- 1654834	- 648966	0	0	0	0	0	0	+ 551627	- 2548833	+ 505665	- 2594795
+ .979373a		-.20310262	+.78824114	+.00967481	+.01839330	+.06144930	+.10772128	+.23955935	+.00988567	-.02401994	+.01620700	+.00635580	0	0	0	0	0	0	-.00540249		-.00495235	
0		+ 93465060	+ 6021595	- 14922869	- 3817702	- 31257740	+ 592864	- 23973794	+ 2553626	+ 6636016	- 1585558	+ 8889217	0	0	0	0	0	0	+ 1357219	+ 64695950	+ 1415663	+ 64754394
+ .2031026	-1.	+ 89253115	+ 22368152	- 14722233	- 3436261	- 29983403	+ 2826790	- 19005808	+ 2758635	+ 6137890	- 1249457	+ 9021024	0	0	0	0	0	0	+ 1245182	+ 65213625	+ 1312961	+ 65281404
- .227558a	+1.120409a		-.25061480	+.16494923	+.03850018	+.33593677	-.03167161	+.21294280	-.03090800	-.06876948	+.01399903	-.10107237	0	0	0	0	0	0	-.01395113		-.01471053	
0	0		+103171103	+ 18420496	- 6755351	+ 3121460	+ 4841829	- 2269319	+ 659529	+ 3967630	- 1422588	+ 3567857	0	0	0	0	0	0	- 808893	+ 52031123	- 203704	+ 52636312
- .8391417	+ .2506148	-1.	+ 34124335	+ 21331436	- 7374544	+ 5690045	- 4536470	- 16786931	- 827466	+ 4362610	- 2413864	+ 795513	0	0	0	0	0	0	- 686139	+ 33678528	- 134166	+ 34230502
+2.459071a	- .734417a	+2.930460a		-.62510921	+.21610806	-.16674449	+.13293944	+.49193430	+.02424856	-.12784454	+.07073732	-.02331219	0	0	0	0	0	0	+.02010703		+.00393168	
0	0	0		+ 95952459	- 20287849	- 34881048	- 818756	- 9616529	+ 2769659	+ 4524292	- 518091	+ 8132501	0	0	0	0	0	0	- 1480881	+ 46285526	- 909585	+ 46856821
+ .5483820	- .3216108	+ .6251092	-1.	+ 80180004	- 16262932	- 43444390	+ 2376897	- 2494508	+ 3732185	+ 2833353	+ 768730	+ 9116951	0	0	0	0	0	0	- 841240	+ 35965047	- 604252	+ 36202032
- .683939a	+ .401111a	- .779632a	+1.247194a		+.20283027	+.54183572	-.02964451	+.03111135	-.04654758	-.03533740	-.00958755	-.11370604	0	0	0	0	0	0	+.01049189		+.00753619	
0	0	0	0		+ 61961305	+ 26268577	- 26605859	- 6997181	- 12103798	- 7093661	- 11182254	+ 672633	- 3322719	+ 4957878	+ 84896	+ 767479	+ 424942	+ 4256017	+ 249928	- 400788	+ 124486	- 526230
- .0806906	- .0495727	- .0893170	- .2028303	-1.	+ 56902151	+ 17416632	- 27197596	- 12312570	- 11437978	- 5294755	- 11626530	+ 3029118	- 3322719	+ 4957878	+ 84896	+ 767479	+ 424942	+ 4256017	- 10895	+ 16636068	+ 32781	+ 16679744
+ .141806a	+ .087119a	+ .156966a	+ .356454a	+1.757403a		-.30608038	+.47797132	+.21638145	+.20101135	+.09305017	+.20432496	-.05323380	+.05839356	-.08712989	-.00149196	-.01348770	-.00746794	-.07479536	+.00019147		-.00057609	
0	0	0	0	0		+223621393	+ 10520246	- 11904100	- 569872	- 10812386	- 45176318	-172853142	+ 3919189	+ 13070049	+ 771316	+ 6972851	+ 3494473	+ 6818684	+ 225239	- 14925475	+ 2045659	- 13105055
+ .4685334	- .5368124	+ .5327892	- .4797534	+ .3060804	-1.	+183343916	+ 21162958	- 14575778	+ 5955976	- 6171342	- 41320068	-165982438	+ 4936208	+ 11552540	+ 745331	+ 6737941	+ 3364407	+ 5516001	+ 339369	+ 15605021	+ 2202736	+ 17468386
- .255549a	+ .292790a	- .290595a	+ .261669a	- .166943a	+ .545423a		-.11542765	+.07949965	-.03248527	+.03365992	+.22536918	+.90530650	-.02692322	-.06301022	-.00406521	-.03675028	-.01835025	-.03008554	-.00185100		-.01201423	
0	0	0	0	0	0		+ 50720183	+ 1342556	- 8871228	- 11687523	- 7695446	+ 3923900	- 1769318	- 4947531	- 304313	- 2751054	- 1376307	- 2481950	- 337558	- 8704265	- 27685	- 8394393
- .3346149	+ .1127909	- .2556601	- .0119257	- .5133015	+ .1154276	-1.	+ 33329829	- 7050717	- 15442482	- 12940153	- 8965490	+ 24010564	- 3927257	- 3911290	- 349767	- 3161966	- 1561543	- 1084395	- 428230	- 1482898	- 253309	- 1307978
+1.003950a	- .338408a	+ .767061a	+ .035781a	+1.540066a	- .346319a	+3.000315a		+.21154375	+.46332317	+.38824541	+.26899298	-.72039265	+.11783010	+.11735104	+.01049411	+.09486895	+.04685122	+.03253527	+.01284825		+.00760007	
0	0	0	0	0	0	0		+126376771	+ 33906916	- 96825616	- 166314	- 10806373	- 5678886	+ 9887513	+ 210231	+ 1900529	+ 1024917	+ 8158841	+ 447005	- 9443304	+ 221613	- 9668696
- .6430492	- .1292054	- .5035396	- .1156631	- .3006339	- .0550816	- .2115438	-1.	+102819696	+ 28693365	- 97070505	- 9689644	- 15826712	- 6836221	+ 11051320	+ 213863	+ 1933367	+ 1054001	+ 9288887	+ 414630	+ 26046048	+ 666159	+ 26297576
+ .625414a	+ .125662a	+ .489731a	+ .112491a	+ .292389a	+ .053571a	+ .205742a	+ .972576a		-.27906487	+.94408473	+.09423918	+.15392685	+.06648747	-.10748252	-.00207998	-.01880347	-.01025096	-.09034151	-.00403259		-.00647890	
0	0	0	0	0	0	0	0		+ 98810357	- 32794619	- 33869364	+ 1341694	- 38651700	+ 29868426	+ 2981470	- 14349817	- 7922027	- 22881136	+ 176400	+ 45130	+ 195616	+ 64346
- .0690596	+ .1477444	- .0665404	+ .0481134	- .3648826	+ .1013369	- .4042887	+ .2790649	-1.	+ 80866511	- 12756516	- 36386071	+ 22193576	- 39391798	+ 25593502	+ 8225299	- 16418976	- 8963536	- 25299444	- 161445	- 7971613	- 183326	- 7993493
+ .085399a	- .182702a	+ .082284a	+ .059497a	+ .451216a	- .125314a	+ .499946a	- .345093a	+1.236606a		+.15774782	+.44995228	-.27444706	+.48712128	-.31649074	-.03403864	+.20303802	+.11084361	+.31285440	+.00199644		+.00226702	
0	0	0	0	0	0	0	0	0		+188893192	- 4279594	- 18903465	+ 42682013	- 51681342	- 8758610	+ 9181735	+ 17585691	- 44500341	+ 2870182	- 8543826	+ 3257754	- 8156253
- .6416831	- .0294711	- .4697620	- .1059206	- .6234178	- .0248617	- .6517363	- .9000629	- .1577478	-1.	+ 88374869	- 24713646	- 27355993	+ 28346304	- 37878992	- 8225299	+ 7487533	+ 16713304	- 39561097	+ 3098877	+ 6285859	+ 3772661	+ 6959642
+ .726092a	+ .033348a	+ .531556a	+ .119854a	+ .705424a	+ .028132a	+ .737468a	+1.018460a	+ .178499a	+1.131543a		+.27964563	+.30954493	-.32075073	+.42861727	+.09307283	-.08472469	-.18911829	+.44765098	-.03506514		-.04268930	
0	0	0	0	0	0	0	0	0	0		+ 89186230	+ 62559084	- 34105645	- 36994280	- 6203776	+ 15732156	+ 7211884	+ 3451712	+ 1201996	- 5510999	+ 406038	- 6306957
- .3500010	- .0478975	- .2524368	- .1620568	- .6402656	- .1608665	- .6530945	- .2203720	- .4940658	- .2796456	-1.	+ 50668082	+ 33158557	- 45170244	- 32465158	- 7154022	+ 11445258	+ 8376867	- 16298489	+ 1979983	+ 4540823	+ 1899203	+ 4460043
+ .690772a	+ .094532a	+ .498217a	+ .319840a	+1.263647a	+ .317491a	+1.288966a	+ .434932a	+ .975103a	+ .551917a	+1.973629a		-.65442692	+.89149307	+.64074180	+.14119386	-.22588674	-.16532828	+.32167172	-.03907752		-.03748322	
0	0	0	0	0	0	0	0	0	0	0		+300927745	- 59646074	-100849543	- 26580699	- 20042486	+ 9710311	- 8865872	- 216621	- 19688299	- 2162739	- 21634417
+ .5502309	- .4710088	+ .5840439	- .2589702	+ .9800055	- .9271704	+1.0244472	- .3649079	+ .5489470	- .1265373	+ .6544269	-1.	+ 92538502	- 4077481	- 83639359	- 24245331	- 12074197	+ 16172160	+ 3475826	+ 160606	- 11689273	+ 32265	- 11817614
- .594597a	+ .508987a	- .631136a	+ .279851a	-1.059025a	+1.001929a	-1.107050a	+ .394331a	- .593209a	+ .136740a	+ .707194a	+1.080631a		+.04406254	+.90383308	+.26200263	+.13047755	-.17476142	-.03756086	-.00173556		-.00034867	
0	0	0	0	0	0	0	0	0	0	0	0		+127452807	+ 7222082	- 13566115	+ 42199305	- 30192084	- 39380086	+ 2256073	- 581157	+ 2941689	+ 104459
- .2151076	+ .0342258	- .1642115	- .1064951	- .6524962	- .0900652	- .6568774	+ .1456080	- .8527918	+ .0658730	- .8626574	- .0440625	-1.	+ 57479306	- 536507	- 17075176	+ 41090359	- 31918398	- 52801733	+ 2923017	- 839132	+ 3293904	- 468244
+ .374235a	- .059545a	+ .285688a	+ .185276a	+1.135185a	+ .156692a	+1.142807a	- .253322a	+1.483650a	- .114603a	+1.500814a	+ .076658a	+1.739757a		+.00933392	+.29706650	-.71487222	+.55530242	+.91862162	-.05085338		-.05730591	
0	0	0	0	0	0	0	0	0	0	0	0	0		+276240736	+ 11321552	-116197373	- 20402190	- 54804650	+ 3919080	- 29389592	+ 3293887	- 30014785
+ .0252264	- .4502080	+ .1826407	- .3405890	+ .3576206	- .9021741	+ .2553275	- .8362797	+ .4205052	- .7215516	- .0573011	- .9042444	- .0093339	-1.	+152695505	- 19850505	-112058002	+ 8739162	- 73392620	+ 6624257	- 37242201	+ 6002764	- 37863693
- .016521a	+ .294840a	- .119611a	+ .223051a	- .234205a	+ .590832a	- .167214a	+ .547678a	- .275388a	+ .472543a	+ .037526a	+ .592188a	+ .006113a	+ .654898a		+.13000059	+.73386575	-.05723261	+.48064689	-.04338213		-.03931199	
0	0	0	0	0	0	0	0	0	0	0	0	0	0		+101907269	+ 20292486	- 46390363	- 26870759	+ 10560688	+ 19455272	+ 10528912	+ 19423497
- .0272087	- .1825917	+ .0472148	- .1757729	- .0310982	- .4100452	- .0427002	- .2833740	- .1052468	- .2399436	- .2334481	- .3926444	- .2982799	- .1300006	-1.	+ 86025266	+ 17574002	- 47488472	- 56368418	+ 12899013	+ 12641392	+ 12908718	+ 12651098
+ .031629a	+ .212254a	- .054885a	+ .204327a	+ .036150a	+ .476657a	+ .049637a	+ .329408a	+ .122344a	+ .278922a	+ .271372a	+ .456429a	+ .346735a	+ .151119a	+1.162449a		-.20428884	+.55202935	+.65525421	-.14994497		-.15005729	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+180401014	- 13102039	- 29595964	+ 1371659	+ 82780481	+ 2305300	+ 83714122
+ .3332604	- .3021734	+ .3681341	- .0949379	+ .9456758	- .5271410	+ .8291164	- .5060293	+ .9332876	- .3962124	+ .9336041	- .6823596	+ .7689576	- .7073081	+ .2042888	-1.	+ 56477860	+ 22514851	- 32381838	+ 725892	+ 47336767	+ 819090	+ 47429965
- .590073a	+ .535030a	- .651820a	+ .168098a	-1.674419a	+ .933359a	-1.468038a	+ .895978a	-1.652484a	+ .701536a	-1.653044a	+1.208189a	-1.361520a	+1.252363a	- .361715a	+1.770605a		-.39864915	+.57335455	-.01285268		-.01450285	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+ 89559693	+ 23674187	- 3336660	+ 29964429	- 3182068	+ 30119021
- .2104428	+ .1934502	- .2171268	+ .0095313	- .7840343	+ .2142942	- .7707761	+ .4856237	-1.0230399	+ .3608350	- .9258472	+ .2573164	-1.0259718	+ .2674362	- .6334689	- .3986492	-1.	+ 27630187	- 13168744	+ 3748808	+ 18210252	+ 3990160	+ 18451604
+ .761641a	- .700141a	+ .785832a	- .034496a	+2.837600a	- .775580a	+2.789616a	-1.757584a	+3.702617a	-1.305945a	+3.350854a	- .931287a	+3.713228a	- .967913a	+2.292670a	+1.442803a	+3.619230a		+.47660713	-.13567798		-.14441307	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+389991549	- 44779124	+162191109	- 45816018	+161154215
- .5154902	- .3151330	- .2309597	- .4616974	- .8393018	- .9426808	-1.0808777	- .7950081	-1.1657415	-1.0315691	-1.2251969	- .9634152	-1.1666572	- .8439066	- .8400401	- .3833555	- .4766071	-1.	+212075907	- 26348222	+185727686	- 26971054	+185104856
+ .243069a	+ .148594a	+ .108904a	+ .217704a	+ .395755a	+ .444502a	+ .509666a	+ .374870a	+ .549681a	+ .486415a	+ .577716a	+ .454278a	+ .550113a	+ .397927a	+ .396103a	+ .180763a	+ .224734a	+ .471529a		+.12423958		+.12717642	
																			+ 6811674	- 18960966	+ 7096453	- 17963064
																			+ 412651	+ 412653	+ 424181	+ 424180

The suffix "a" indicates that values must be multiplied by 10^{-8}

TABLE XVIII - Forward solution of normal equations

TABLE XVIII

Forward solution of normal equations

	$Qd\eta_3$	$Qd\xi_3$	$Qd\eta_4$	$Qd\xi_4$	$Qd\eta_5$	$Qd\xi_5$	$Qd\eta_6$	$Qd\xi_6$	$Qd\eta_7$	$Qd\xi_7$	$Qd\eta_8$	$Qd\xi_8$	$Qd\eta_9$	$Qd\xi_9$	$Qd\eta_{10}$	$Qd\xi_{10}$	$Qd\eta_{11}$	$Qd\xi_{11}$	
$Qd\eta_3$	+ 5.938202																		$Qd\eta_3$
$Qd\xi_3$	- 1.728086	+ 2.439079																	$Qd\xi_3$
$Qd\eta_4$	+ 4.977148	- 1.814822	+ 5.286009																$Qd\eta_4$
$Qd\xi_4$	- .573971	+ .986662	- .890831	+ 1.846747															$Qd\xi_4$
$Qd\eta_5$	+ 4.042434	- 1.690461	+ 3.637195	+ .356269	+10.104231														$Qd\eta_5$
$Qd\xi_5$	- .981109	+ 1.765058	- 1.366326	+ 1.186595	- 2.347304	+ 3.401614													$Qd\xi_5$
$Qd\eta_6$	+ 4.294688	- 1.751620	+ 3.702010	+ .178657	+ 8.964512	- 2.105171	+10.416254												$Qd\eta_6$
$Qd\xi_6$	+ .655795	+ 1.457494	+ .042632	+ .901922	- 1.568169	+ 2.358128	- 1.374486	+ 4.418483											$Qd\xi_6$
$Qd\eta_7$	+ 2.810301	- 1.604952	+ 2.586125	+ .111132	+ 7.765574	- 1.725044	+ 7.729069	- 2.794960	+ 9.437046										$Qd\eta_7$
$Qd\xi_7$	+ .557162	+ 1.009689	+ .043252	+ .746099	- .589386	+ 1.882263	- .288675	+ 3.056789	- 1.314699	+ 2.969585									$Qd\xi_7$
$Qd\eta_8$	+ 2.963031	- 1.189157	+ 2.607561	+ .434863	+ 7.606302	- 1.102751	+ 7.579072	- 1.937830	+ 8.300268	- .812350	+ 9.150110								$Qd\eta_8$
$Qd\xi_8$	- .945076	+ 1.544709	- 1.290314	+ .903254	- 2.698030	+ 2.994943	- 2.416931	+ 2.432534	- 2.279577	+ 1.951863	- 1.934200	+ 3.300419							$Qd\xi_8$
$Qd\eta_9$	+ 1.902257	- .949864	+ 1.702717	+ .337638	+ 5.804347	- .690471	+ 5.741584	- 2.204810	+ 7.228333	- 1.338830	+ 6.965109	- 1.136205	+ 7.341650						$Qd\eta_9$
$Qd\xi_9$	- .428336	+ 1.013506	- .706039	+ .561458	- 1.838732	+ 1.895507	- 1.515050	+ 2.010631	- 1.954633	+ 1.764751	- 1.505009	+ 2.138514	- 1.440633	+ 2.155018					$Qd\xi_9$
$Qd\eta_{10}$	+ .838838	- .215739	+ .667558	+ .331014	+ 2.508197	+ .168076	+ 2.544815	- .652099	+ 3.267175	- .283061	+ 3.217037	+ .001280	+ 3.439210	- .383593	+ 3.021421				$Qd\eta_{10}$
$Qd\xi_{10}$	- .800519	+ .871105	- .923342	+ .265308	- 2.653911	+ 1.412946	- 2.384733	+ 1.740346	- 2.917806	+ 1.408620	- 2.767389	+ 1.753596	- 2.630906	+ 1.790768	- 1.123838	+ 2.415074			$Qd\xi_{10}$
$Qd\eta_{11}$	+ .877489	- .629320	+ .837736	+ .069263	+ 3.026220	- .563727	+ 3.032526	- 1.578918	+ 3.964599	- 1.074116	+ 3.626198	- .714775	+ 3.975416	- .778258	+ 2.481456	- 1.356650	+ 3.726340		$Qd\eta_{11}$
$Qd\xi_{11}$	+ .243069	+ .148594	+ .108904	+ .217704	+ .395755	+ .444502	+ .509666	+ .374870	+ .549681	+ .486415	+ .577716	+ .454278	+ .550113	+ .397927	+ .396103	+ .180763	+ .224734	+ .471529	$Qd\xi_{11}$

all values must be multiplied by 10^{-8} to correspond to dt expressed in micro-degrees

TABLE XIX - Weight and correlate numbers

TABLE XIX

Weight and correlate numbers

TABLE XX

Backward solution of normal equations

	solution 1	solution 2		solution 1	solution 2
$d\eta_3$	+.00190457	-.01441978	$d\xi_3$	-.00624177	-.00450165
$d\eta_4$	+.00123433	-.01670020	$d\xi_4$	+.00962389	+.00304986
$d\eta_5$	-.05058771	-.06515905	$d\xi_5$	+.02169006	+.02013833
$d\eta_6$	-.02215786	-.04063546	$d\xi_6$	+.04139532	+.04292180
$d\eta_7$	-.04477276	-.05687723	$d\xi_7$	+.06188327	+.06374290
$d\eta_8$	-.08070067	-.08832197	$d\xi_8$	+.04375713	+.05244688
$d\eta_9$	-.08035734	-.09107229	$d\xi_9$	+.06916601	+.07686880
$d\eta_{10}$	-.12890037	-.13174242	$d\xi_{10}$	+.08886316	+.09182108
$d\eta_{11}$	-.07646451	-.08379988	$d\xi_{11}$	+.12423958	+.12717642

Standard error of weight unit

$$\mu^2 = \frac{412652}{58 - 31} = 15283.4 \qquad \mu^2 = \frac{424180}{58 - 31} = 15710.4$$

$$\mu = \pm 123.6 \qquad \mu = \pm 125.3$$

$$\text{p.e.} = \pm 0''30 \pm 0''03 \qquad \text{p.e.} = \pm 0''30 \pm 0''03$$

TABLE XXI

Standard errors of unknowns

	m_η	m_ξ
3	±.03	±.02
4	±.03	±.02
5	±.04	±.02
6	±.04	±.03
7	±.04	±.02
8	±.04	±.02
9	±.03	±.02
10	±.02	±.02
11	±.02	±.01

TABLE XXII

Preliminary and final positions

Preliminary positions

Station	η	ξ	
Pedro	953 141.52	2937 707.52	Fixed
Palo	954 443.61	2947 231.53	Fixed
Eltoro	943 738.58	2950 586.28	
Fordyce	943 041.28	2942 909.36	
Garcia	929 677.35	2947 352.92	
Pancho	930 885.24	2958 524.00	
Corpus	923 095.82	2957 697.42	
Monument	923 627.84	2948 018.77	
Grande	917 511.34	2951 740.37	
Ringold	910 967.31	2949 419.24	
Hebron	911 111.47	2957 835.25	
Garcena	906 653.16	2957 386.92	Fixed
Gorgora	898 718.52	2953 935.72	Fixed

Final positions

Station	η	ξ
Eltoro	943 738.57 ± .03	2950 586.28 ± .02
Fordyce	943 041.26 ± .03	2942 909.36 ± .02
Garcia	929 677.28 ± .04	2947 352.94 ± .02
Pancho	930 885.20 ± .04	2958 524.04 ± .03
Corpus	923 095.76 ± .03	2957 697.48 ± .02
Monument	923 627.75 ± .04	2948 018.82 ± .02
Grande	917 511.25 ± .03	2951 740.44 ± .02
Ringold	910 967.18 ± .02	2949 419.33 ± .02
Hebron	911 111.39 ± .02	2957 835.38 ± .01

Standard error of an observation = ± 125 micro-degrees

APPENDIX II

COEFFICIENTS FOR EQUATIONS H AND I

The tables given in this appendix are arranged in the form of "Nutshell Tables" or "Taylor Tables" after Hirvonen (2). In these, functions of the various derivatives of the tabulated functions are given, so that the value desired may be arrived at by use of a Taylor's series, through the formula ——

$$f(a+th) = f(a) + th\, f'(a) + \tfrac{1}{2} t^2h^2\, f''(a) + \ldots ,$$

where "a" denotes the values tabulated, "h" the unit of the last decimal given for "a", and "t" the fraction of the interval to be covered, or decimals of "h".

The following expressions are tabulated ——

$f(a)$,

$A = h\, f'(a)$,

$B = \tfrac{1}{2} h^2\, f''(a)$, etc.

In the terminology of the tables, the interpolation formula now reads

$$f(a+th) = \left\{ \left[(Dt + C)t + B \right] t + A \right\} t + f(a).$$

The interval of tabulation in these tables is "2h", which allows interpolation through less than "h" by interpolating either up or down, depending on which way is the shorter.

The units of the tabulated factors are such that when they are used with ξ and η in meters, the scale factor is obtained as a pure number, and the (T-t) correction is obtained in degrees.

TABLE XXIII

Factor H_2 as a function of ξ, International Ellipsoid

$\xi x10^{-5}$	$H_2 x10^{12}$	A	B	C
0	0.012382 97847	00000	-04151	0
2	12382 81247	- 16594	4143	+ 3
4	12382 31517	33119	4117	6
6	12381 48862	49509	4075	8
8	12380 33618	65696	4016	11
10	0.012378 86260	- 81611	- 3940	+14
12	12377 07394	97194	3849	17
14	12374 97750	1 12378	3741	19
16	12372 58189	1 27102	3618	22
18	12369 89690	1 41306	3481	24
20	0.012366 93350	-1 54933	- 3330	+26
22	12363 70380	1 67928	3166	28
24	12360 22092	1 80241	2988	30
26	12356 49904	1 91820	2799	32
28	12352 55330	2 02621	2600	34
30	0.012348 39965	-2 12602	- 2389	+36
32	12344 05494	2 21723	2170	37
34	12339 53669	2 29950	1942	39
36	12334 86309	2 37253	1708	40
38	12330 05291	2 43603	1466	41
40	0.012325 12548	-2 48975	- 1219	+41
42	12320 10052	2 53354	969	42
44	12314 99805	2 56723	715	43
46	12309 83841	2 59071	459	43
48	12304 64207	2 60390	- 201	43
50	0.012299 42966	-2 60679	+ 56	+43
52	12294 22175	2 59941	313	43
54	12289 03885	2 58179	568	42
56	12283 90136	2 55401	820	42
58	12278 82945	2 51625	1067	41
60	0.012273 84290	-2 46866	+ 1312	+40
62	12268 96124	2 41142	1549	39
64	12264 20345	2 34482	1780	38
66	12259 58800	2 26914	2003	37
68	12255 13276	2 18466	2219	35
70	0.012250 85499	-2 09174	+ 2425	+34
72	12246 77115	1 99077	2622	32
74	12242 89700	1 88214	2808	30
76	12239 24739	1 76629	2983	28
78	12235 83636	1 64366	3146	26
80	0.012232 67694	-1 51475	+ 3297	+24
82	12229 78121	1 38006	3435	22
84	12227 16021	1 24010	3561	20
86	12224 82397	1 09541	3671	17
88	12222 78133	94657	3769	15
90	0.012221 04008	- 79412	+ 3852	+13

TABLE XXIV

Factors H_4 and H_6 as functions of ξ, International Ellipsoid

$\xi x10^{-5}$	$H_4 x10^{24}$	A	$H_6 x10^{36}$
0	0.0000 26248	0	0.0000 00021
2	26247	- 1	21
4	26243	3	21
6	26235	4	21
8	26226	6	21
10	0.0000 26213	- 7	0.0000 00021
12	26198	8	21
14	26180	10	21
16	26159	11	21
18	26137	12	21
20	0.0000 26111	- 13	0.0000 00021
22	26084	14	21
24	26054	15	21
26	26023	16	21
28	25989	17	21
30	0.0000 25954	- 18	0.0000 00021
32	25918	19	21
34	25880	19	21
36	25840	20	21
38	25800	20	21
40	0.0000 25759	- 21	0.0000 00021
42	25717	21	21
44	25674	21	21
46	25631	22	20
48	25588	22	20
50	0.0000 25545	- 22	0.0000 00020
52	25502	21	20
54	25460	21	20
56	25417	21	20
58	25376	21	20
60	0.0000 25335	- 20	0.0000 00020
62	25295	20	20
64	25256	19	20
66	25219	18	20
68	25182	18	20
70	0.0000 25148	- 17	0.0000 00020
72	25115	16	20
74	25083	15	20
76	25054	14	20
78	25027	13	20
80	0.0000 25001	- 12	0.0000 00020
82	24978	11	20
84	24957	10	20
86	24938	9	20
88	24922	8	20
90	0.0000 24908	- 6	0.0000 00020

TABLE XXV

Factor I_1 as a function of ξ_r, International Ellipsoid

$\xi_r x10^{-5}$	$I_1 x10^{12}$	A	B	C
0	0.7094 92404	0	- 2378	0
2	7094 82893	- 9508	2374	2
4	7094 54400	18976	2359	3
6	7094 07042	28366	2335	5
8	7093 41012	37641	2301	7
10	0.7092 56582	- 46760	- 2257	8
12	7091 54100	55688	2205	10
14	7090 33982	64388	2143	11
16	7088 96724	72824	2073	12
18	7087 42886	80962	1995	14
20	0.7085 73095	- 88770	- 1908	15
22	7083 88047	96216	1814	16
24	7081 88493	1 03270	1712	17
26	7079 75245	1 09905	1604	19
28	7077 49170	1 16093	1490	20
30	0.7075 11184	-1 21812	- 1369	20
32	7072 62250	1 27038	1243	21
34	7070 03373	1 31752	1113	22
36	7067 35596	1 35936	979	23
38	7064 59993	1 39574	840	23
40	0.7061 77672	-1 42652	- 699	24
42	7058 89763	1 45161	555	24
44	7055 97413	1 47091	410	24
46	7053 01788	1 48436	263	25
48	7050 04059	1 49193	- 115	25
50	0.7047 05410	-1 49358	+ 32	25
52	7044 07018	1 48935	179	24
54	7041 10060	1 47926	326	24
56	7038 15704	1 46334	470	24
58	7035 25105	1 44170	612	23
60	0.7032 39397	-1 41443	+ 752	23
62	7029 59698	1 38164	887	22
64	7026 87097	1 34349	1020	22
66	7024 22651	1 30012	1148	21
68	7021 67384	1 25172	1271	20
70	0.7019 22286	-1 19848	+ 1389	19
72	7016 88299	1 14063	1502	18
74	7014 66327	1 07839	1609	17
76	7012 57220	1 01201	1709	16
78	7010 61782	94175	1802	15
80	0.7008 80761	- 86789	+ 1889	14
82	7007 14848	79072	1968	13
84	7005 64676	71052	2040	11
86	7004 30819	62763	2103	10
88	7003 13784	54235	2159	9
90	0.7002 14018	- 45500	+ 2207	7

TABLE XXVI

Factor I_2 as a function of ξ_r, International Ellipsoid

$\xi_r x10^{-5}$	$I_2 x10^{18}$	A	B	C
00	.0000 00000	+23785	000	-4
2	47538	23736	- 24	4
4	94880	23590	49	4
6	1 41832	23347	73	4
8	1 88203	23008	97	4
10	.0002 33800	+22575	-120	-4
12	2 78440	22049	143	4
14	3 21938	21434	165	4
16	3 64119	20732	186	4
18	4 04811	19946	207	3
20	.0004 43851	+19080	-226	-3
22	4 81080	18137	245	3
24	5 16352	17122	262	3
26	5 49524	16039	279	3
28	5 80468	14894	294	2
30	.0006 09060	+13690	-308	-2
32	6 35191	12433	320	2
34	6 58761	11129	332	2
36	6 79680	9783	341	2
38	6 97869	8401	350	1
40	.0007 13263	+06988	-356	-1
42	7 25807	5552	362	-1
44	7 35457	4096	366	0
46	7 42182	2628	368	0
48	7 45964	+01153	369	0
50	.0007 46793	-00323	-369	0
52	7 44676	1794	367	0
54	7 39626	3254	363	+1
56	7 31672	4697	358	1
58	7 20853	6119	352	1
60	.0007 07216	-07513	-345	+1
62	6 90824	8874	336	2
64	6 71745	10198	326	2
66	6 50060	11479	315	2
68	6 25859	12713	302	2
70	.0005 99241	-13896	-289	+2
72	5 70314	15022	274	2
74	5 39194	16088	259	3
76	5 06004	17091	242	3
78	4 70875	18026	225	3
80	.0004 33944	-18892	-207	+3
82	3 95357	19683	188	3
84	3 55262	20399	169	3
86	3 13814	21036	149	3
88	2 71172	21592	129	3
90	.0002 27500	-22066	-108	+4

TABLE XXVII

Factors I_3 and I_4 as functions of ζ_r, International Ellipsoid

$\zeta_r x10^{-5}$	$I_3 x10^{24}$	A	B	$I_4 x10^{30}$
00	0.0057 78336	00000	- 19	0.0000 00000
2	57 78259	- 77	19	196
4	57 78028	153	19	392
6	57 77645	229	19	585
8	57 77111	305	19	777
10	0.0057 76428	-00378	- 18	0.0000 00965
12	57 75599	451	18	1149
14	57 74627	521	17	1328
16	57 73516	589	17	1502
18	57 72271	655	16	1669
20	0.0057 70897	-00719	- 15	0.0000 01830
22	57 69399	779	15	1983
24	57 67783	836	14	2128
26	57 66057	890	13	2264
28	57 64226	940	12	2390
30	0.0057 62298	-00987	- 11	0.0000 02507
32	57 60282	1029	10	2614
34	57 58184	1068	9	2710
36	57 56014	1102	8	2795
38	57 53780	1131	7	2868
40	0.0057 51491	-01157	- 6	0.0000 02931
42	57 49156	1177	5	2981
44	57 46785	1193	3	3019
46	57 44386	1204	2	3046
48	57 41971	1211	- 1	3060
50	0.0057 39547	-01212	0	0.0000 03062
52	57 37124	1209	+ 1	3052
54	57 34713	1201	3	3030
56	57 32322	1189	4	2996
58	57 29962	1171	5	2951
60	0.0057 27641	-01149	+ 6	0.0000 02894
62	57 25368	1123	7	2825
64	57 23152	1092	8	2746
66	57 21002	1057	9	2657
68	57 18926	1018	10	2557
70	0.0057 16933	-00975	+ 11	0.0000 02447
72	57 15030	928	12	2328
74	57 13224	877	13	2201
76	57 11523	824	14	2065
78	57 09932	766	15	1921
80	0.0057 08459	-00706	+ 15	0.0000 01770
82	57 07108	644	16	1612
84	57 05886	578	17	1448
86	57 04796	511	17	1279
88	57 03843	442	18	1105
90	0.0057 03031	-00371	+ 18	0.0000 00927

TABLE XXVIII

Factor H_2 as a function of ξ, Clarke 1866 Ellipsoid

$\xi x10^{-5}$	$H_2 x10^{12}$	A	B	C
0	0.012384 25700	00000	- 4180	0
2	12384 08984	- 16711	4172	+ 3
4	12383 58900	33355	4147	6
6	12382 75655	49860	4103	9
8	12381 59597	66160	4044	11
10	0.012380 11195	- 82190	- 3968	+14
12	12378 31061	97883	3876	17
14	12376 19931	1 13174	3767	19
16	12373 78675	1 28001	3644	22
18	12371 08274	1 42307	3506	24
20	0.012368 09836	-1 56029	- 3353	+26
22	12364 84583	1 69115	3188	29
24	12361 33835	1 81514	3009	31
26	12357 59021	1 93173	2819	33
28	12353 61664	2 04050	2618	34
30	0.012349 43372	-2 14099	- 2405	+36
32	12345 05843	2 23283	2185	37
34	12340 50838	2 31568	1955	39
36	12335 80194	2 38919	1719	40
38	12330 95802	2 45310	1476	41
40	0.012325 99607	-2 50719	- 1228	+42
42	12320 93592	2 55128	976	42
44	12315 79775	2 58518	719	43
46	12310 60207	2 60879	461	43
48	12305 36950	2 62205	- 202	43
50	0.012300 12077	-2 62494	+ 58	+43
52	12294 87665	2 61747	315	43
54	12289 65775	2 59970	573	43
56	12284 48464	2 57172	826	42
58	12279 37760	2 53365	1076	41
60	0.012274 35663	-2 48570	+ 1320	+40
62	12269 44126	2 42806	1561	39
64	12264 65070	2 36097	1792	38
66	12260 00349	2 28474	2018	37
68	12255 51766	2 19964	2235	35
70	0.012251 21059	-2 10605	+ 2443	+34
72	12247 09887	2 00436	2640	32
74	12243 19829	1 89495	2829	30
76	12239 52391	1 77827	3003	28
78	12236 08973	1 65479	3169	26
80	0.012232 90896	-1 52497	+ 3320	+24
82	12229 99373	1 38933	3459	22
84	12227 35517	1 24838	3586	20
86	12225 00336	1 10269	3697	17
88	12222 94720	95281	3795	15
90	0.012221 19452	- 79930	+ 3878	+13

TABLE XXIX

Factors H_4 and H_6 as functions of ξ, Clarke 1866 Ellipsoid

$\xi x10^{-5}$	$H_4 x10^{24}$	A	$H_6 x10^{36}$
0	0.0000 26258	0	0.0000 00021
2	26257	- 1	21
4	26253	3	21
6	26246	4	21
8	26236	6	21
10	0.0000 26223	- 7	0.0000 00021
12	26208	8	21
14	26190	10	21
16	26169	11	21
18	26146	12	21
20	0.0000 26120	- 13	0.0000 00021
22	26093	14	21
24	26063	15	21
26	26031	16	21
28	25998	17	21
30	0.0000 25962	- 18	0.0000 00021
32	25925	19	21
34	25887	19	21
36	25848	20	21
38	25807	21	21
40	0.0000 25765	- 21	0.0000 00021
42	25723	21	21
44	25680	22	21
46	25637	22	20
48	25594	22	20
50	0.0000 25550	- 22	0.0000 00020
52	25507	22	20
54	25464	21	20
56	25421	21	20
58	25380	21	20
60	0.0000 25338	- 20	0.0000 00020
62	25298	20	20
64	25259	19	20
66	25221	19	20
68	25185	18	20
70	0.0000 25150	- 17	0.0000 00020
72	25117	16	20
74	25085	15	20
76	25056	14	20
78	25028	13	20
80	0.0000 25003	- 12	0.0000 00020
82	24979	11	20
84	24958	10	20
86	24939	9	20
88	24923	8	20
90	0.0000 24909	- 6	0.0000 00020

TABLE XXX

Factor I_1 as a function of ξ_r, Clarke 1866 Ellipsoid

$\xi_r x10^{-5}$	$I_1 x10^{12}$	A	B	C
0	0.7095 65658	0	-2395	0
2	7095 56081	- 9575	2390	2
4	7095 27385	19111	2376	3
6	7094 79689	28568	2351	5
8	7094 13193	37907	2317	6
10	0.7093 28165	- 47091	-2274	8
12	7092 24955	56083	2221	10
14	7091 03987	64844	2158	11
16	7089 65757	73339	2088	13
18	7088 10829	81536	2009	14
20	0.7086 39837	- 89398	-1921	15
22	7084 53480	96896	1827	17
24	7082 52517	1 04000	1724	18
26	7080 37764	1 10680	1615	19
28	7078 10095	1 16912	1500	20
30	0.7075 70431	-1 22670	-1378	21
32	7073 19746	1 27932	1252	21
34	7070 59047	1 32679	1120	22
36	7067 89388	1 36891	985	23
38	7065 11852	1 40552	846	23
40	0.7062 27553	-1 43651	- 704	24
42	7059 37628	1 46178	559	24
44	7056 43232	1 48120	412	25
46	7053 45542	1 49473	264	25
48	7050 45738	1 50232	- 116	25
50	0.7047 45008	-1 50398	+ 33	25
52	7044 44542	1 49970	180	25
54	7041 45521	1 48952	328	25
56	7038 49123	1 47349	473	24
58	7035 56511	1 45167	617	23
60	0.7032 68831	-1 42420	+ 756	23
62	7029 87201	1 39118	894	22
64	7027 12722	1 35274	1027	22
66	7024 46457	1 30906	1156	21
68	7021 89438	1 26030	1281	20
70	0.7019 42661	-1 20668	+1400	19
72	7017 07076	1 14841	1513	18
74	7014 83590	1 08573	1621	17
76	7012 73063	1 01887	1721	16
78	7010 76299	94812	1816	15
80	0.7008 94054	- 87374	+1902	14
82	7007 27024	79603	1982	13
84	7005 75846	71527	2055	11
86	7004 41097	63179	2118	10
88	7003 23288	54592	2174	9
90	0.7002 22867	- 45797	+2222	7

TABLE XXXI

Factor I_2 as a function of ξ_r, Clarke 1866 Ellipsoid

$\xi_r x10^{-5}$	$I_2 x10^{18}$	A	B	C
00	+.0000 00000	+23954	000	-4
2	47875	23905	- 25	4
4	95553	23757	49	4
6	1 42838	23512	73	4
8	1 89537	23171	97	4
10	+.0002 35458	+22731	-121	-4
12	2 80413	22205	144	4
14	3 24219	21586	166	4
16	3 66697	20878	188	4
18	4 07676	20087	208	3
20	+.0004 46989	+19214	-228	-3
22	4 84480	18264	247	3
24	5 19998	17242	264	3
26	5 53402	16151	281	3
28	5 84560	14997	296	2
30	+.0006 13350	+13784	-310	-2
32	6 39661	12518	323	2
34	6 63392	11205	334	2
36	6 84452	9849	344	2
38	7 02764	8457	352	1
40	+.0007 18260	+07034	-359	-1
42	7 30885	5587	364	1
44	7 40597	4121	368	1
46	7 47362	2643	371	-1
48	7 51163	+01157	372	0
50	+.0007 51991	-00329	-371	0
52	7 49851	1810	369	0
54	7 44759	3280	366	+1
56	7 36742	4734	361	1
58	7 25840	6165	355	1
60	+.0007 12101	-07569	-347	+1
62	6 95587	8940	338	1
64	6 76368	10272	328	2
66	6 54526	11562	317	2
68	6 30150	12805	304	2
70	+.0006 03341	-13995	-291	+2
72	5 74207	15129	276	3
74	5 42865	16203	261	3
76	5 09439	17212	244	3
78	4 74062	18154	227	3
80	+.0004 36872	-19024	-209	+3
82	3 98013	19821	190	3
84	3 57637	20542	170	3
86	3 15899	21183	150	3
88	2 72960	21743	130	3
90	+.0002 28983	-22220	-109	+4

TABLE XXXII

Factors I_3 and I_4 as functions of ξ_r, Clarke 1866 Ellipsoid

$\xi_r x10^{-5}$	$I_3 x10^{24}$	A	B	$I_4 x10^{30}$
00	0.0057 78990	00000	- 19	0.0000 00000
2	57 78912	- 77	19	198
4	57 78681	154	19	395
6	57 78295	231	19	590
8	57 77757	307	19	782
10	0.0057 77068	-00381	- 18	0.0000 00972
12	57 76233	453	18	1157
14	57 75256	524	18	1338
16	57 74137	594	17	1512
18	57 72881	661	16	1681
20	0.0057 71498	-00723	- 15	0.0000 01843
22	57 69991	784	15	1997
24	57 68362	842	14	2143
26	57 66624	896	13	2280
28	57 64781	947	12	2407
30	0.0057 62840	-00993	- 11	0.0000 02525
32	57 60810	1036	10	2632
34	57 58697	1075	9	2729
36	57 56512	1109	8	2814
38	57 54262	1140	7	2889
40	0.0057 51956	-01165	- 06	0.0000 02951
42	57 49605	1186	5	3002
44	57 47217	1202	3	3040
46	57 44801	1212	2	3066
48	57 42371	1219	- 01	3081
50	0.0057 39929	-01221	0	0.0000 03083
52	57 37489	1218	+ 01	3073
54	57 35061	1209	3	3051
56	57 32654	1197	4	3017
58	57 30277	1179	5	2971
60	0.0057 27942	-01157	+ 06	0.0000 02914
62	57 25652	1131	7	2845
64	57 23421	1099	8	2765
66	57 21258	1064	9	2675
68	57 19167	1025	10	2574
70	0.0057 17161	-00981	+ 11	0.0000 02464
72	57 15244	934	12	2344
74	57 13426	884	13	2215
76	57 11712	829	14	2078
78	57 10110	772	15	1934
80	0.0057 08627	-00711	+ 15	0.0000 01781
82	57 07268	648	16	1623
84	57 06038	583	16	1458
86	57 04939	515	17	1287
88	57 03981	445	17	1112
90	0.0057 03162	-00373	+ 18	0.0000 00933

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